Status of the OSC Experiment Preparations

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Contribution came from A. Romanov, M. Andorf, J. Ruan
FNAL

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Fermilab
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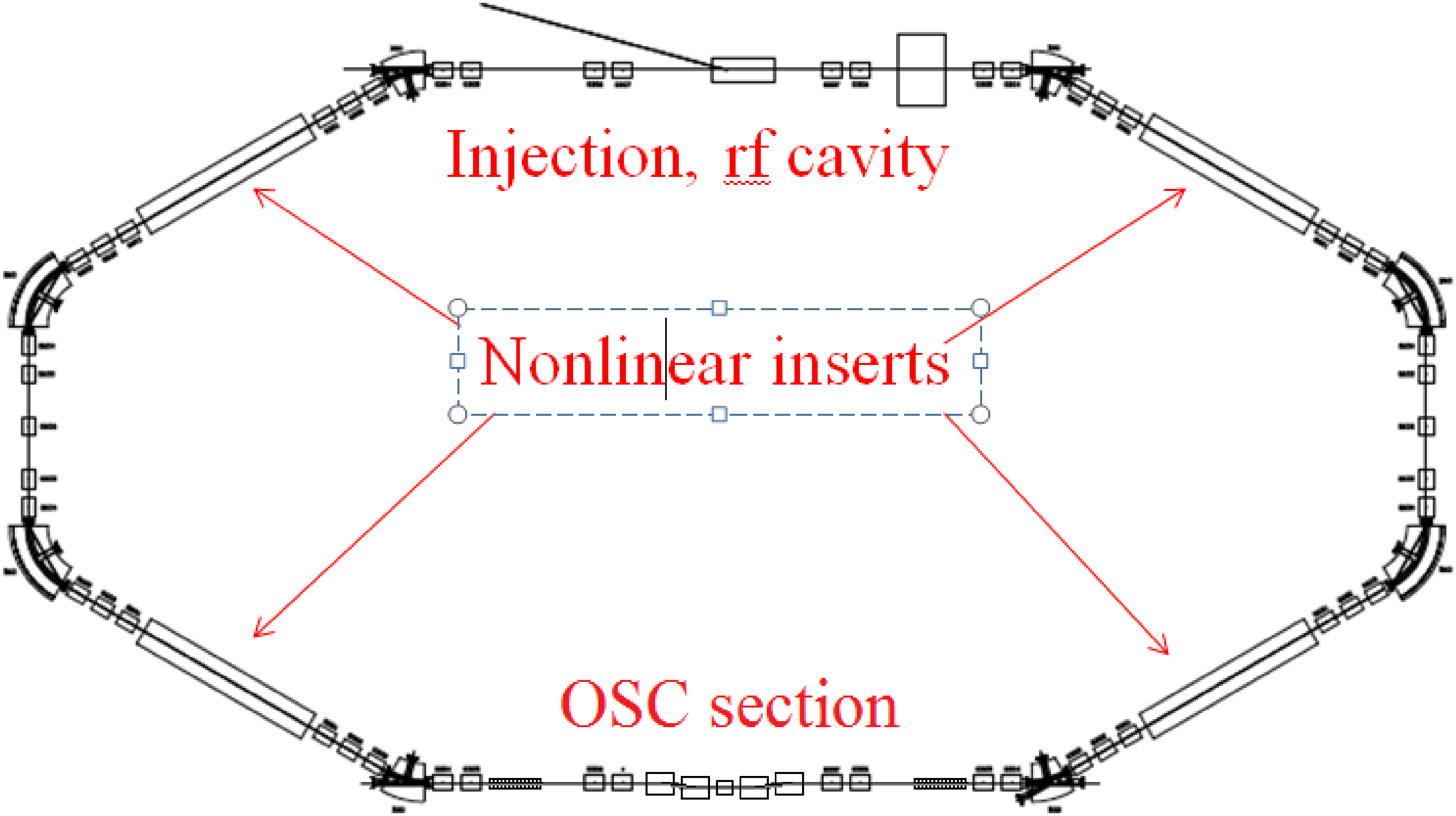
U.S. DEPARTMENT OF ENERGY

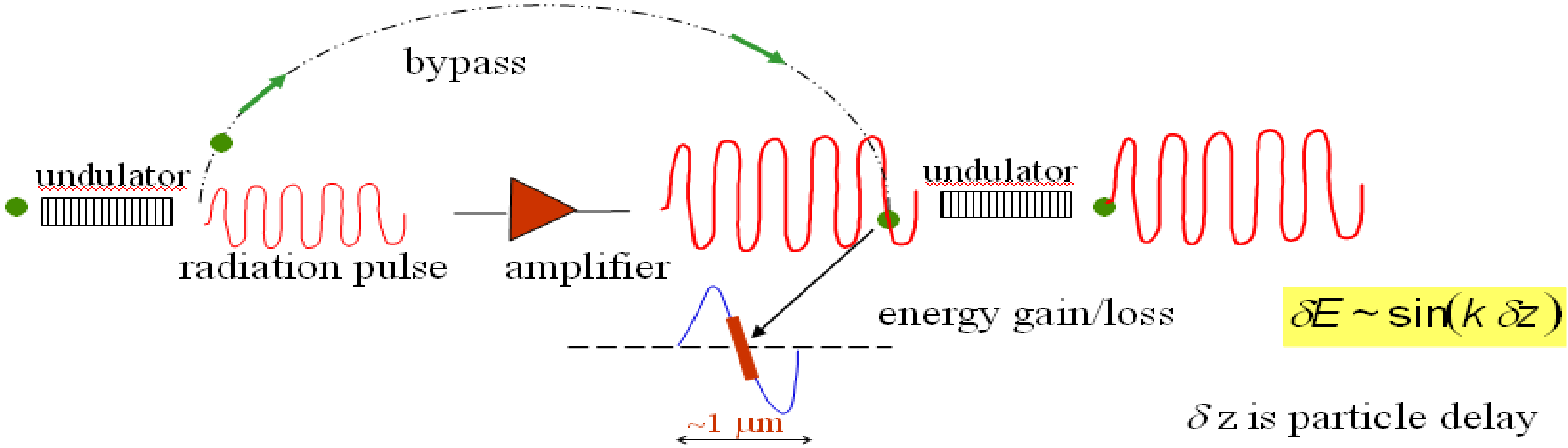
Fermi Research Alliance LLC

Test of OSC in Fermilab

- IOTA - a dual purpose small electron ring
  - Integrable optics
  - OSC
    - ~6 m straight is devoted to OSC



Test of OSC in Fermilab (2)



$\delta E \sim \sin(k\,\delta z)$

δz is particle delay

- Major parameters
  - 100 MeV ($\gamma \approx 200$) electrons
  - Basic wave length – 2.2 μm
    - 7 periods undulators
- Two modes of operation
  - Passive – Optical telescope with suppression of depth of field
  - Active - ~7 dB optical amplifier
- Only longitudinal kicks are effective
  - Requires s-x coupling for horizontal cooling
  - and x-y coupling for vertical cooling

Basics of OSC: Damping Rates

- Linearized longitudinal kick in pickup wiggler

$$\frac{\delta p}{p} = k\xi_0\,\Delta s = k\xi_0\left(M_{51}x_1 + M_{52}\theta_{x_1} + M_{56}\frac{\Delta p}{p}\right) \xrightarrow[\text{betatron motion}]{\text{in the absence of}} k\xi_0\left(M_{51}D_x + M_{52}D_x' + M_{56}\right)\frac{\Delta p}{p}$$

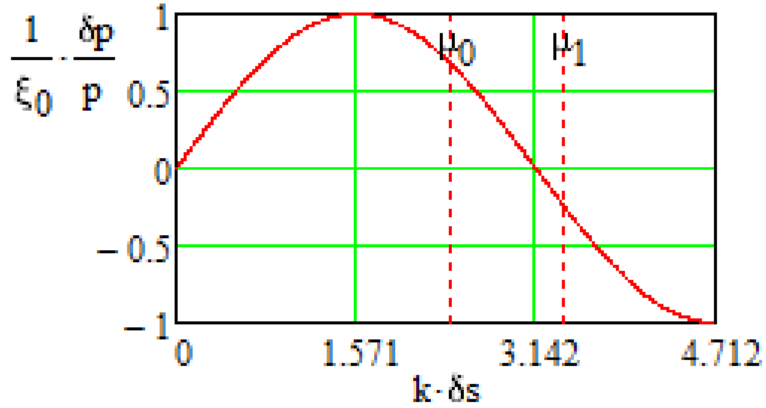
- Partial slip factor (pickup-to-kicker) describes a longitudinal particle displacement in the course of synchrotron motion

$$\tilde{M}_{56} = M_{51}D_1 + M_{52}D_1' + M_{56}$$

- Cooling rates (per turn)

$$\boxed{\begin{aligned}\lambda_x &= \frac{k\xi_0}{2}\left(M_{56} - \tilde{M}_{56}\right)\\ \lambda_s &= \frac{k\xi_0}{2}\tilde{M}_{56}\end{aligned}} \quad \Leftrightarrow \quad \boxed{\lambda_x + \lambda_s = \frac{k\xi_0}{2}M_{56}}$$

Basics of OSC: Cooling Range



- Cooling force depends on Δs nonlinearly

$$\frac{\delta p}{p} = k\xi_0 \Delta s \quad \Rightarrow \quad \frac{\delta p}{p} = \xi_0 \sin\left(k\delta s\right)$$

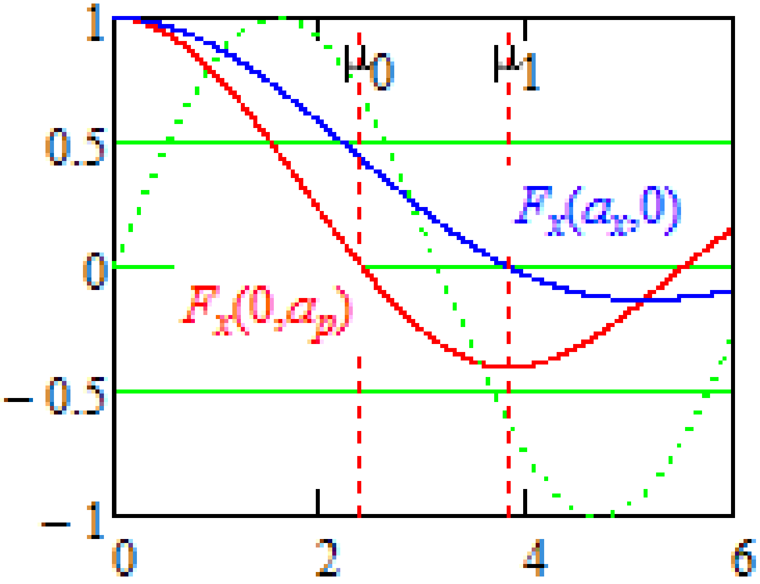
where $k\delta s = a_x \sin(\psi_x) + a_p \sin(\psi_p)$

and a_x & a_p are the amplitudes of longitudinal displacements in cooling chicane due to ⊥ and L motions measured in units of laser phase

$$x = x_0 \sin(\psi_x)\,, \quad \Delta p / p = \left(\Delta p / p\right)_0 \sin(\psi_p)$$

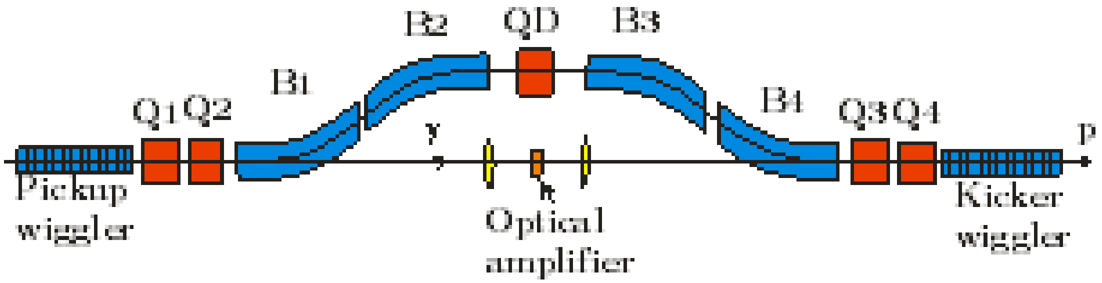
- Averaging yields the form-factors for damping rates

$$\boxed{\begin{aligned} &\lambda_{s,x}(a_x,a_p) = F_{s,x}(a_x,a_p)\lambda_{s,x} \\ &F_x(a_x,a_p) = \frac{2}{a_x} \mathrm{J}_0(a_p)\mathrm{J}_1(a_x) \\ &F_p(a_x,a_p) = \frac{2}{a_p} \mathrm{J}_0(a_x)\mathrm{J}_1(a_p) \end{aligned}}$$

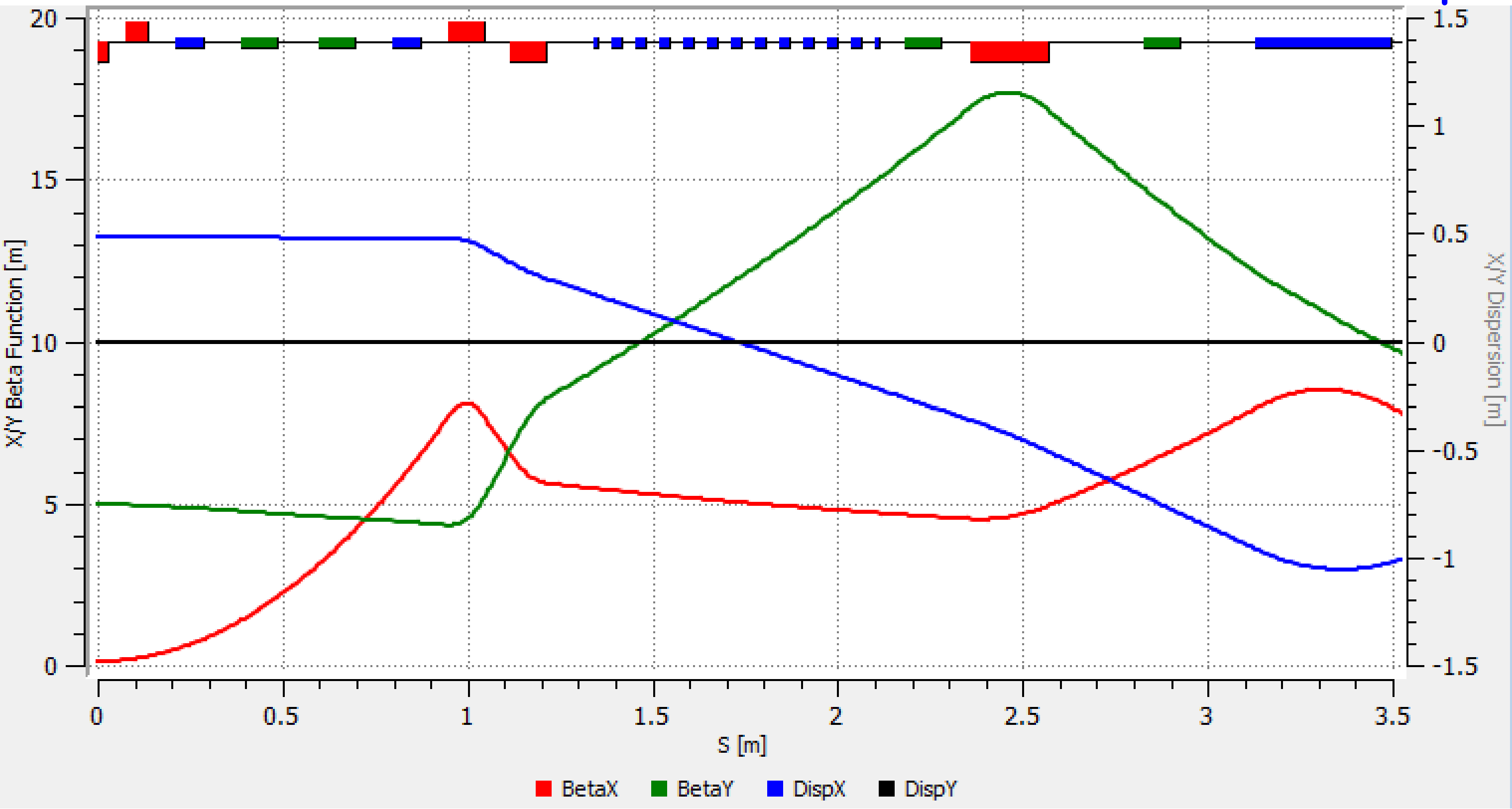


- Damping requires both lengthening amplitudes (a_x and a_p) to be smaller than $\mu_0 \approx 2.405$

Beam and Light Optics

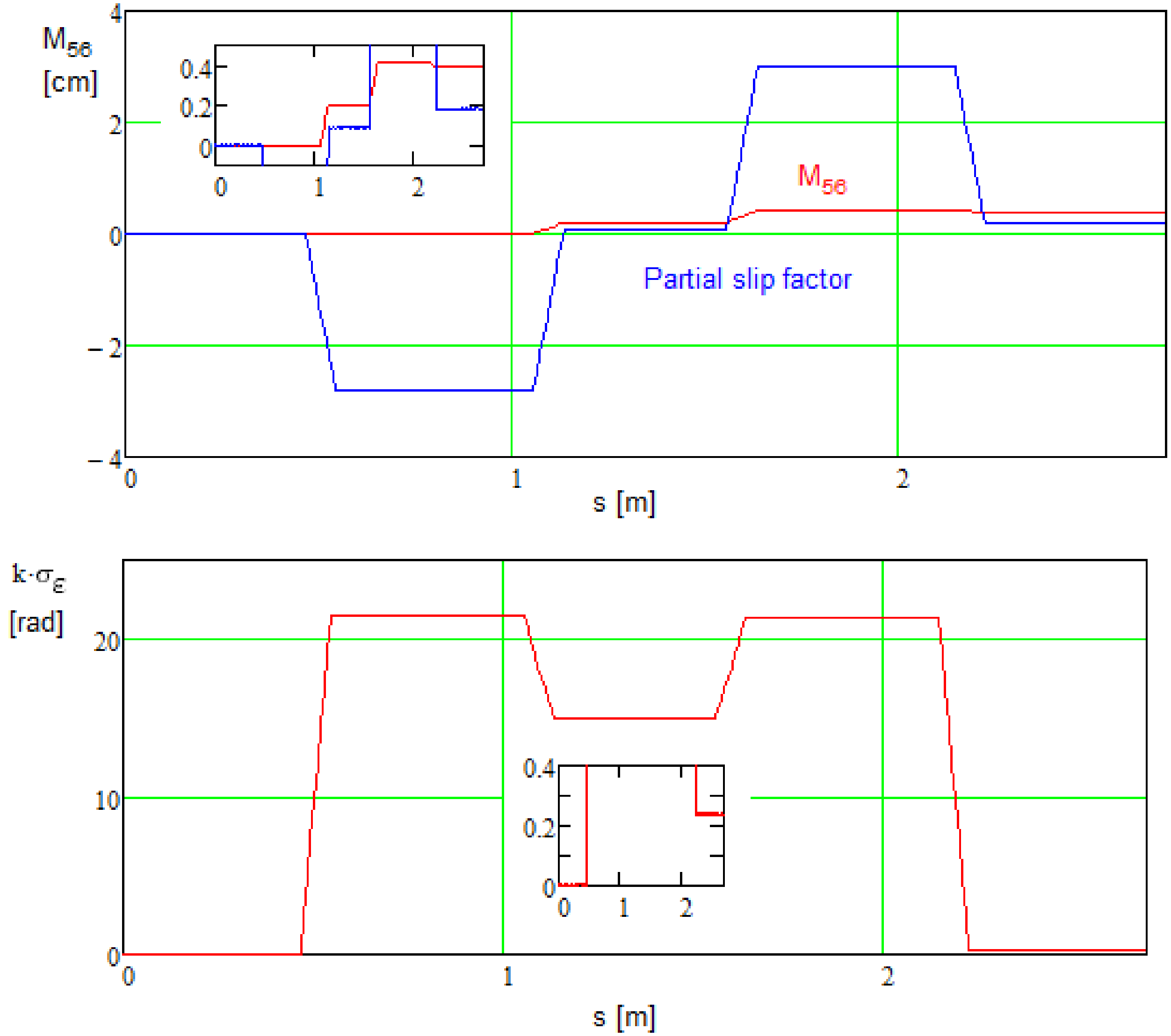


- Chicane to separate beams: optical amplifier & light focusing
- Collider type optics is required to maximize cooling range for x-plane
  - Rectangular dipoles
  - QD introduces non-zero M51 & M52 => transverse damping



Optics functions for half OSC straight (starting from center)

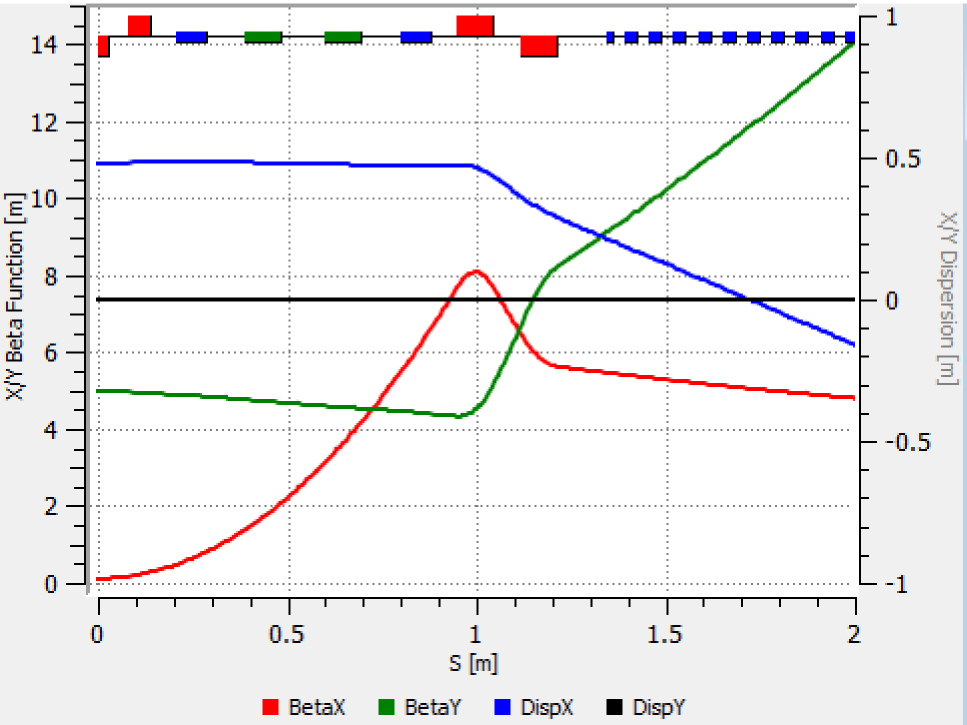
Linear Sample Lengthening on the Travel through Chicane



Sample lengthening due to momentum spread (top) and due to betatron motion (bottom, H. emittance for x-y coupled case)

- Very large sample lengthening on the travel through chicane
- High accuracy of dipole field is required to prevent uncontrolled lengthening, $\Delta(BL)/(BL)_{dipole}<10^{-3}$

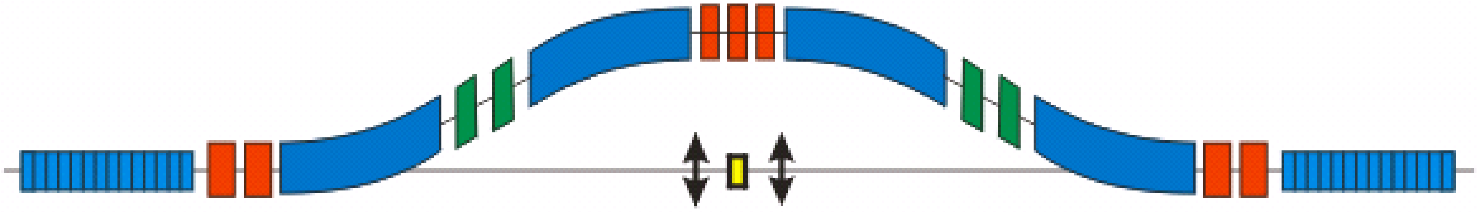
Basics of OSC: Non-linearity of Longitudinal Motion



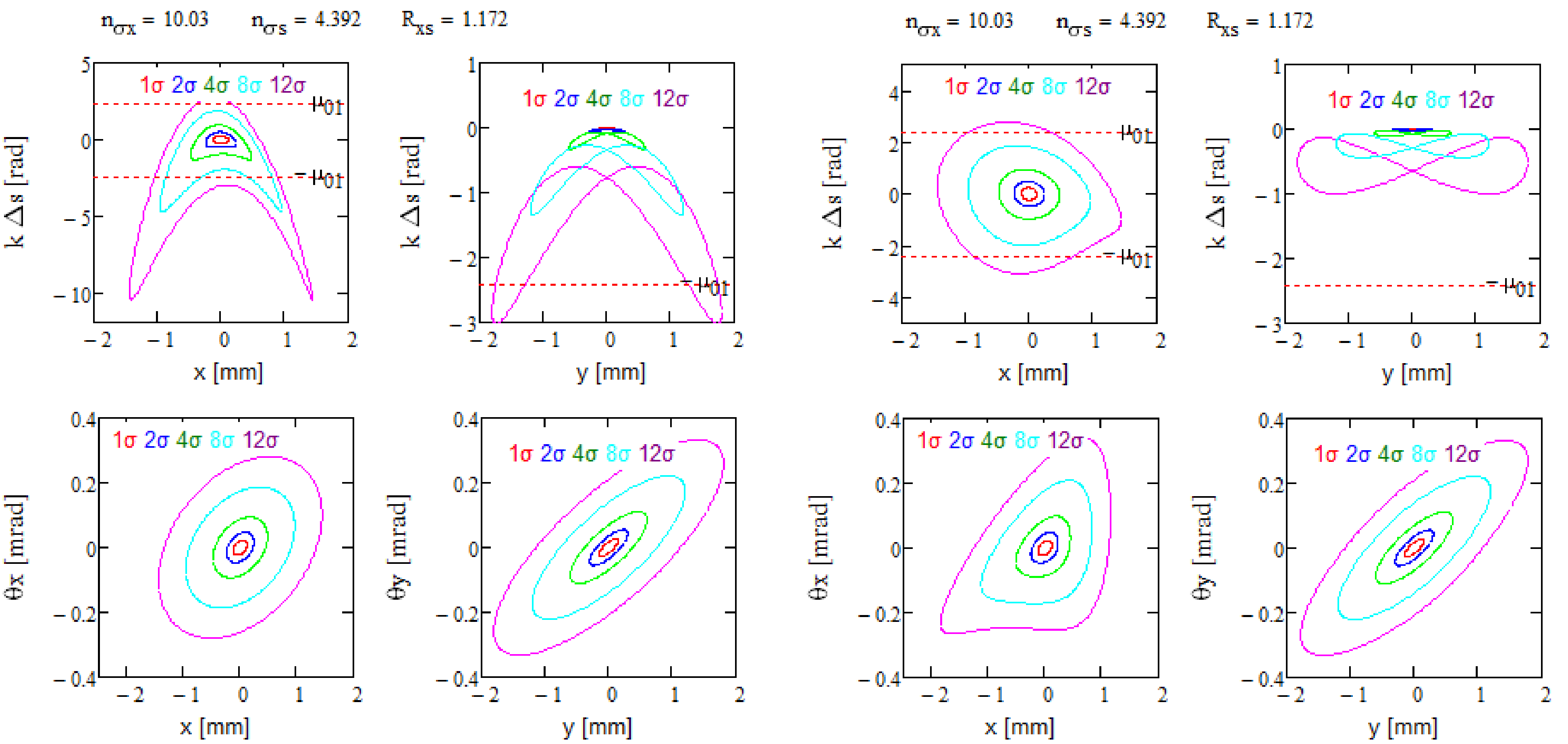
- Major non-linear contribution comes from particle angles

$$\Delta s = M_{51}x_1 + M_{52}\theta_{x_1} + M_{56}\frac{\Delta p}{p} + \frac{1}{2}\int_{s_1}^{s_2}\left(\theta_x{}^2 + \theta_y{}^2\right)ds + \ldots$$

  - It is large and has to be compensated
  - X-plane makes much larger contribution due to small β_x^*
- Correction of path length non-linearity is achieved by two pairs of sextupoles located between dipoles of each dipole pair of the chicane
  - Very strong sextupoles: SdL_y=-7.5 kG/cm, SdL_x=1.37 kG/cm. It results in considerable limitation of the dynamic aperture.



Compensation of Non Linear Sample Lengthening



Phase space distortion for the cases of uncompensated (left) and compensated (right) sample lengthening ($n_{\sigma x}$ is computed for the reference emittance equal to the horizontal emittance of x-y uncoupled case set by SR)

IOTA Optics

Main Parameters of IOTA storage ring for OSC

Circumference	40 m
Nominal beam energy	100 MeV
Bending field	4.8 kG
SR rms x emittance, $\varepsilon_{xSR}(\varepsilon_y = 0)$	2.6 nm
Rms momentum spread, σ_p	$1.06\cdot10^{-4}$
SR damping times (ampl.), $\tau_x / \tau_y / \tau_s$	1.7/2/1.1 s

Main parameters of cooling chicane

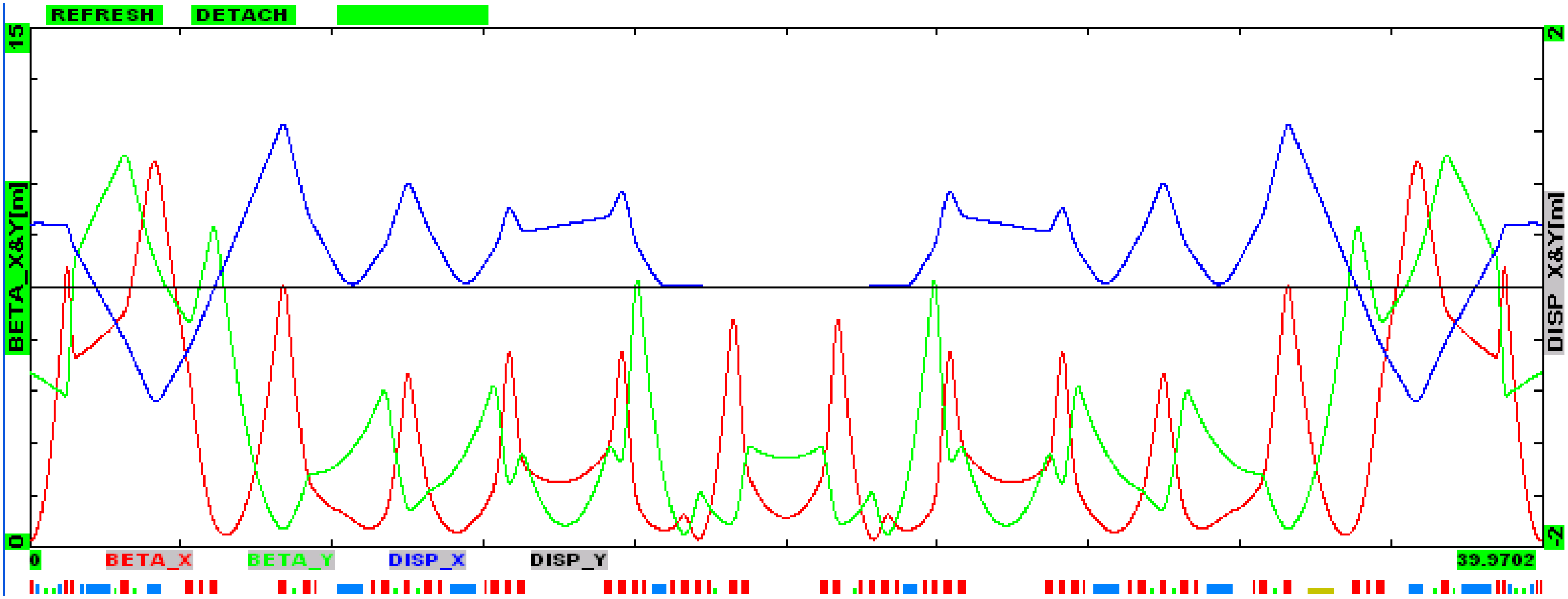
Delay in the chicane, Δs	2 mm
Horizontal beam offset, h	35.1 mm
M_{56}	3.91 mm
D^* / β^*	48 cm / 12 cm
Cooling rates ratio, $(\lambda_x = \lambda_y)/\lambda_s$	0.58
Cooling ranges (before OSC), $n_{\sigma x} = n_{\sigma y} / n_{\sigma s}$	14 / 4.4
Dipole: magnetic field *length	2.5 kG * 8 cm
Strength of central quad, GdL	0.45 kG

- Energy is reduced 150→100 MeV to reduce ε, σ_p and length of undulator period
- Operation on coupling resonance Q_x/Q_y= 5.42/3.42 reduces horizontal emittance and introduces vertical damping

- Small β^* is required to minimize sample lengthening due betatron motion

IOTA Optics (continue)

Alex Romanov

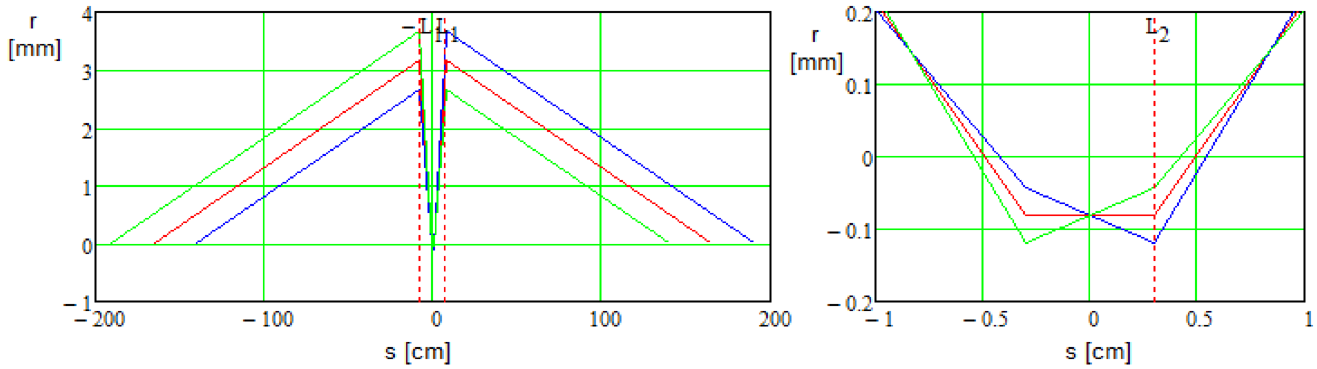


Beta functions ad dispersion through the entire ring

Focusing of Beam Radiation in Passive Scheme

- Three lens system with complete suppression of depth of field

$$\begin{pmatrix}1 & L_1\\ 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ -\dfrac{1}{F_1} & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_2\\ 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ -\dfrac{1}{F_2} & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_2\\ 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ -\dfrac{1}{F_1} & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_1\\ 0 & 1\end{pmatrix} = p\cdot\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$



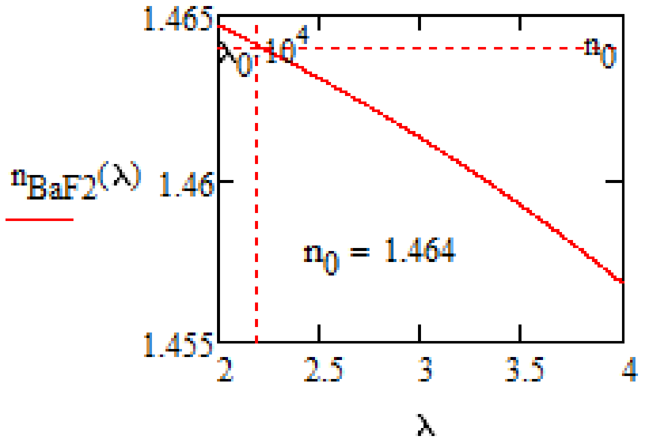
$L_2 := 33$ cm

$L_1 := L_{tot} - L_2 = 140$ cm

$F_1 := L_2 = 33$ cm

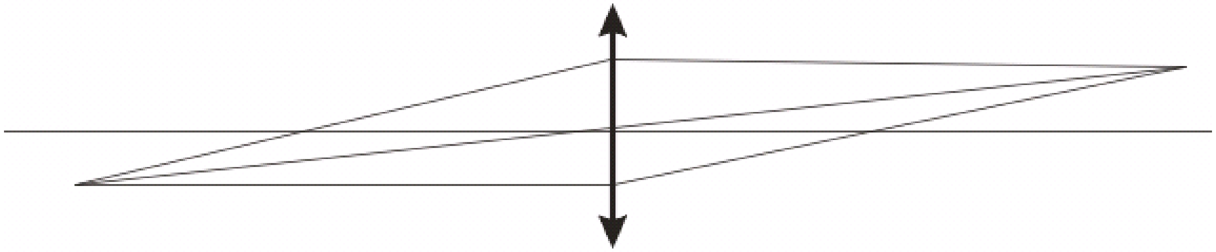
$F_2 := \dfrac{{L_2}^2}{2\cdot(L_2 - L_1)} = -5.089$ cm

- Lenses are manufactured from barium fluoride (BaF$_2$)
  - Excellent material with very small second order dispersion
  - Antireflection coating should protect from humidity damage

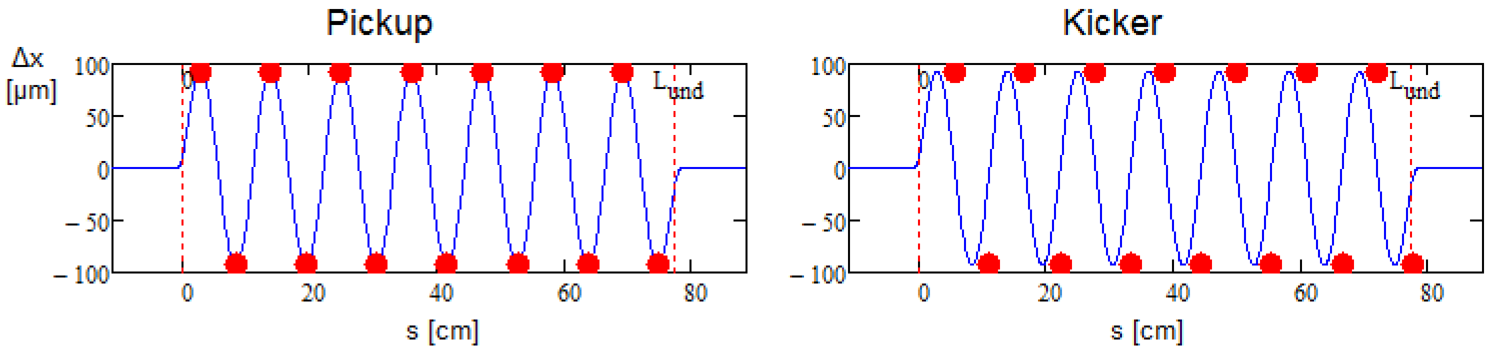


Effect of Beams Overlap on Cooling Rates

- There are 2 possible solutions for three lens telescope
  (1) With positive identity matrix
  (2) With negative identity matrix
- The second choice is preferred for two reasons
  - ♦ Smaller focusing chromaticity
  - ♦ Transfer matrices for particles are close to the negative identity matrix. It mostly compensates separation of light and particles due to betatron motion

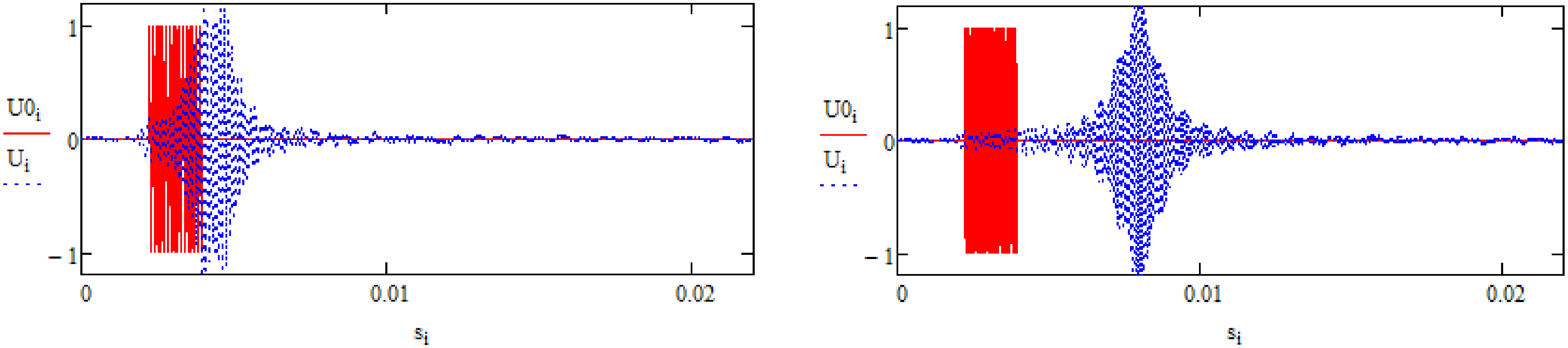


- Particle motion in undulators have to be also accounted


Pickup
Kicker

First Order Dispersion Effects in Optical Lenses

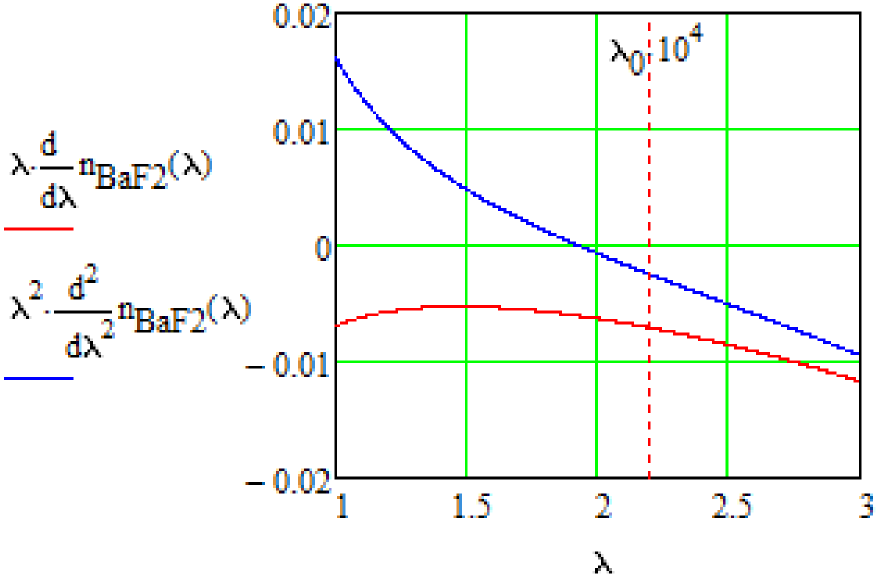
- The first order dispersion, dn/dλ, results in 1.5% difference between phase and group velocities in the lens material
  - It has to be accounted in the total lens thickness
  - Significant separation of radiation of the first and higher harmonics
    - Higher harmonics do not interact resonantly in kicker and have little effect on cooling

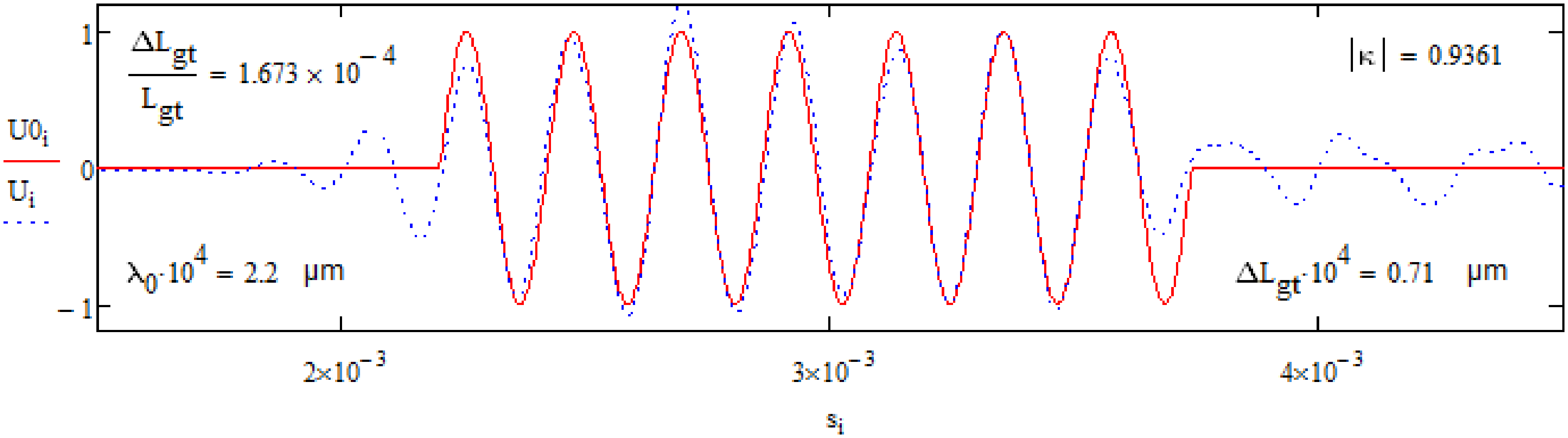


Overlap of radiation for the second and third harmonics of undulator radiation

- Dependence of focusing strength on wave length (1 st order chromaticity) results in a few percent reduction of cooling rates

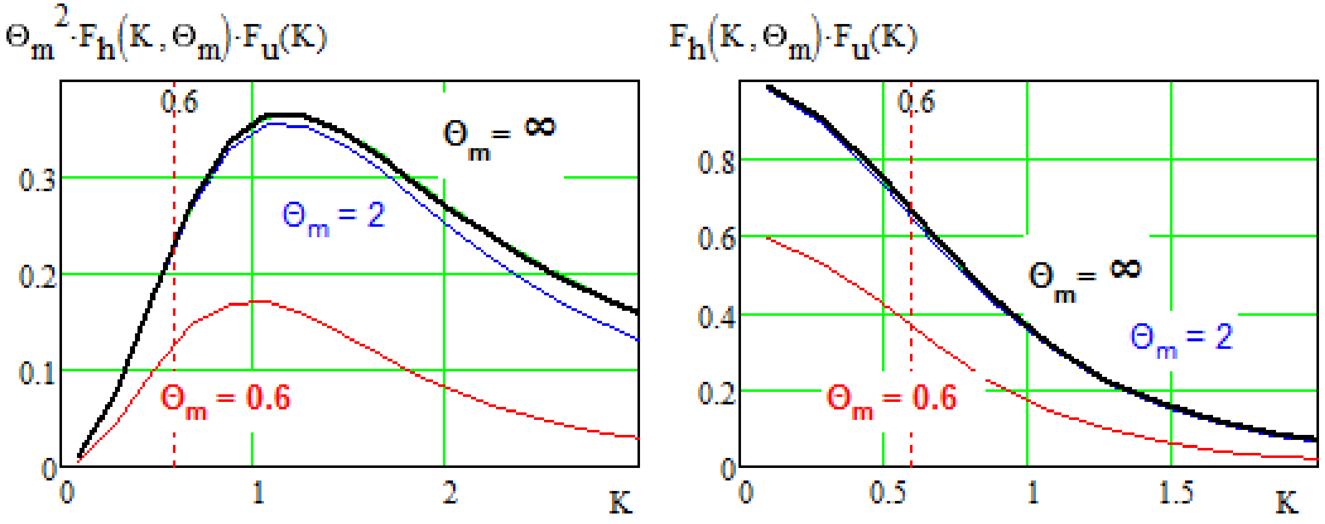
Second Order Dispersion Effects in Optical Lenses





The second order dispersion, $d^2n/d\lambda^2$, results in lengthening of the light packet and, consequently, 6% loss of cooling rates

Dependence of Cooling Efficiency on Undulator Parameter



- With increase of K_U a particle motion in undulator becomes comparable to the size of the focused radiation
  - It reduces cooling efficiency
- An increase of K_U also increases undulator magnetic field and, consequently, the equilibrium emittance and undulator focusing
- Chosen undulator parameter K=1.038 corresponds to the 7 period undulator with B_0=1 kG. It results in a moderate increase of equilibrium emittance of ~5%.

Cooling Rates

- Undulator period was chosen so that $\lambda|_{\theta=0}$=2.2 μm
- Cooling rates were computed using earlier developped formulas(HB2012)
  - Optical system bandwidth of ~40% is limited by telescope acceptance λ=[2.2-3.1]μm
  - Effective bandwidth of SC system is determined by number of undulator periods and dispersion in the lens: $1/n_{per}$
    - Higher harmonics of SR radiation, if present, introduce small additional diffusion ($1/n_{poles}$) and reduce effective bandwidth
- 4 mrad angular acceptance of optical system (aperture a=7 mm)
- Undulator parameter K≈1 is close to the optimal for chosen bandwidth and aperture ($\theta_{max}\gamma$=0.8)

Main parameters of OSC

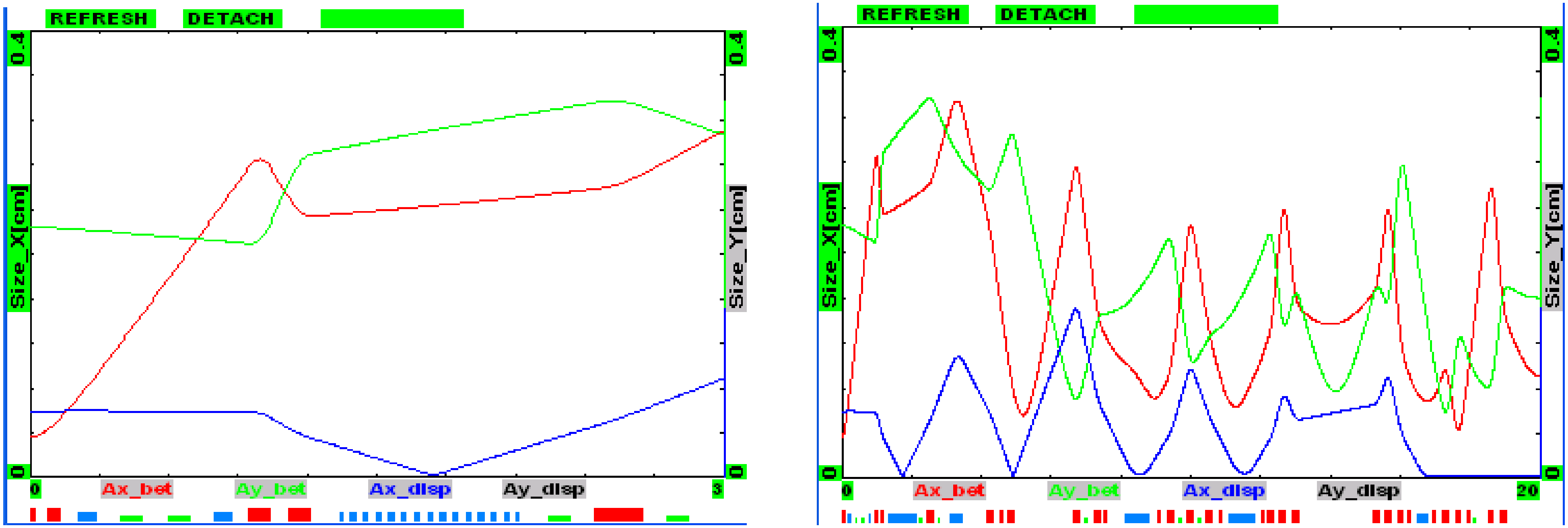
Undulator parameter, K	1.038
Undulator period	11.063 cm
Radiation wavelength at zero angle	2.2 μm
Number of periods, *m*	7
Total undulator length, L_w	0.774 m
Length from OA to undulator center	1.75 m
Telescope aperture, 2*a*	14 mm
OSC damp. rates (*x*=*y*/*s*)	5.8/10 s^{-1}

Beam Parameters and Beam Lifetime

RF voltage, V_{RF}	30 V
Harmonic number	4
RF frequency	30 MHz
SR loses per turn	13.2 eV
Momentum compaction	-0.0165
Bucket height, $\Delta p/p\|_{max}$	$1.08\cdot10^{-3}$ (10σ)
Synchrotron tune	$4.8\cdot10^{-5}$ (360 Hz)
Bunch length set by SR	21 cm
Particles per bunch, N_e	1 - 10^7
Geom. acceptance with OSC insert	1 μm
Dynamic acceptance	0.25μm(10σ for ε_{xSR})
Touschek lifetime @ $N_e=2\cdot10^5$	1.46 hour
Effective vacuum (H_2)	$2\cdot10^{-10}$ Torr
Vacuum lifetime	1.9 hour
$(d\varepsilon_{x,y}/dt)_{gas}$ / $(d\varepsilon_x/dt)_{SR}$	0.027/0.034
$(d\varepsilon_x/dt)_{IBS}$ / $d\varepsilon_x/dt)_{SR}$ @ $N_e=2\cdot10^5$	0.39
$(d\sigma_p^2/dt)_{IBS}/(d\sigma_p^2/dt)_{SR}$ @$N_e=2\cdot10^5$	0.46

- Particle interaction through cooling system is negligible. N_e~10^{10} to get to optimal gain
- Touschek lifetime and IBS growth rates are computed for V_{RF}=30 V and $\varepsilon_x=\varepsilon_y=\varepsilon_{xSR}/2$
- Geometric acceptance should be at least twice larger than the dynamic one
- Vacuum lifetime is computed for dynamic acceptance

Apertures for Electron Beam and its Radiation



Beam sizes at geometric acceptance for half of OSC straight and half of the ring,
ε=1 mm mrad, σ_p=1.2·10^{-3}

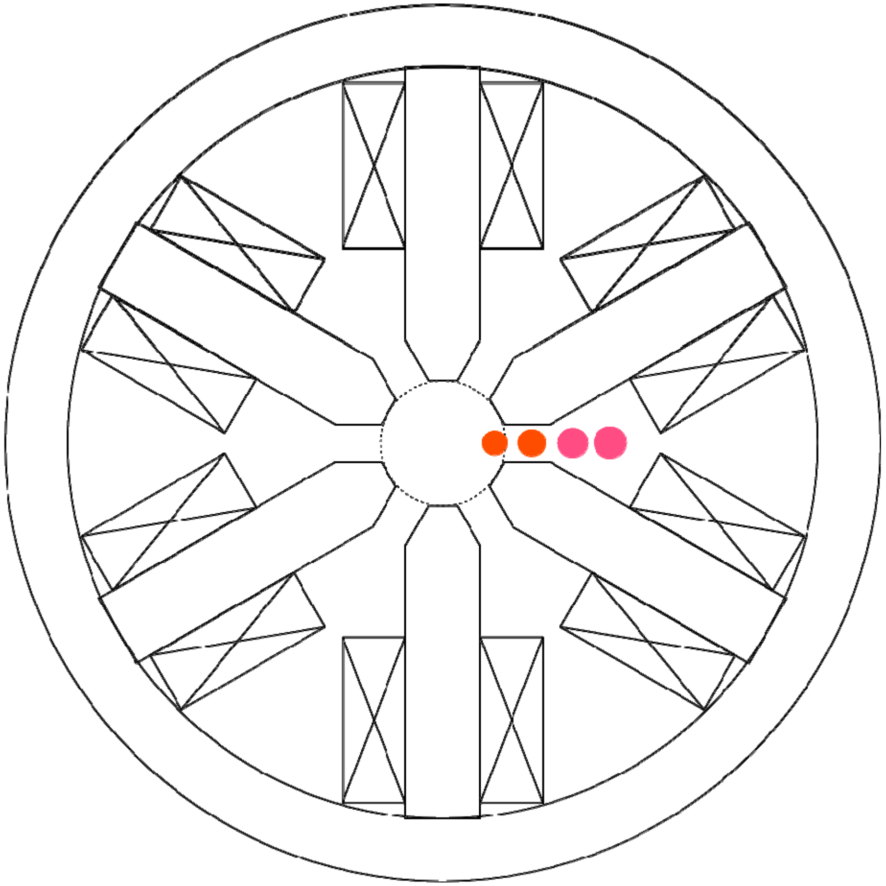
- We require geometric beam acceptance should be at least twice larger than the dynamic one
  - ⇨ Ø8 mm minimum beam aperture in the OSC chicane
- SR divergence ±4 mrad corresponds to SR bandwidth of 40% (2.2-3.1 μm)
  - ♦ It requires aperture of Ø10 mm in the outer sextupoles (tightest place)

Mechanical and Magnetic parameters of Sextupoles

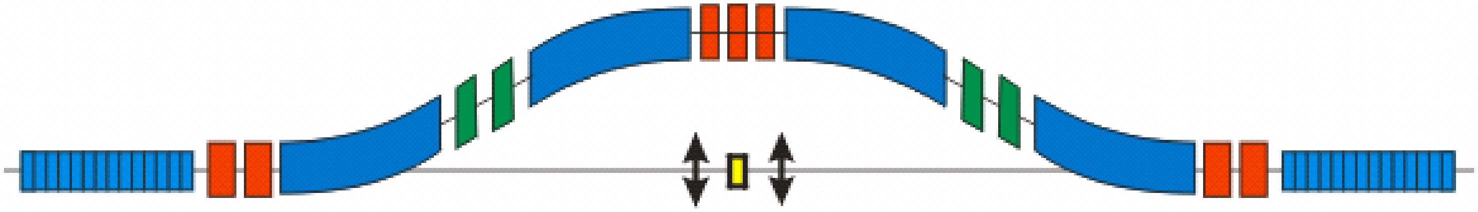
Alex Romanov

Required parameters for sextupoles

Inner diameter	2 cm
Maximum gradient (d2B/dr2)	2 kG/cm^2
Coil current	265 A
Side pole gap for light (full)	6 mm
Outer diameter	120 mm
Field at r=1cm	1 kG



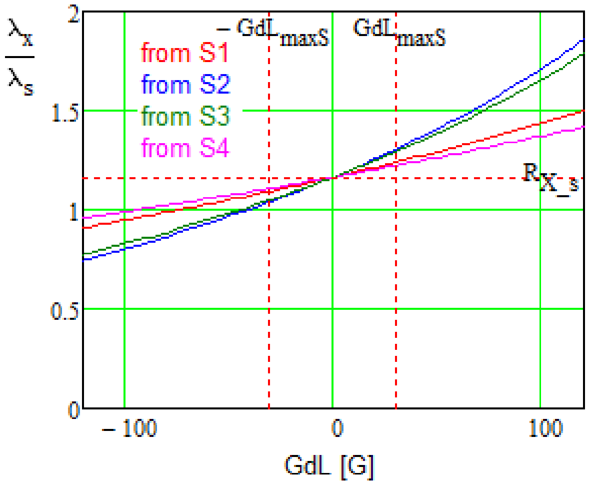
- Tight aperture inside sextupoles requires makes magnetic design and mechanical design of vacuum chamber interdependent



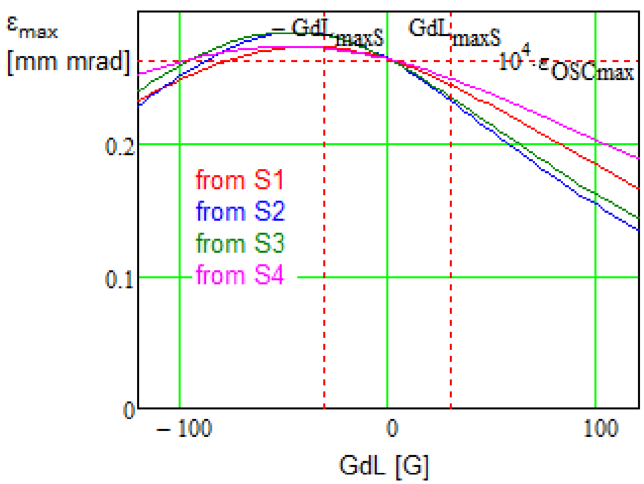
Beam Optics Sensitivity to Errors in Magnets

- Sextupoles are located at larger beta-function than the beta-function in the OSC chicane center and have larger effect on optics
  - Feeddown of quad focusing from sextupoles has to be below GdL~30 G
- Required beam position stability in sextupoles is <20 μm
- Optics measurements correct for feeddown focusing from sextupoles
- Magnetic field of OSC chicane dipoles has to be within $2{\cdot}10^{-4}$ in the good field region of 2a=8 mm

Effect of focusing due to feeddown in sextupoles on the ratio of cooling rates



Effect of focusing due to feeddown in sextupoles on the cooling range



That yields the following limitations:

Maximum offset in a sextupole - $\Delta x_{maxS} := \dfrac{\mathrm{GdL}_{maxS}}{\mathrm{SdL}_x}$ $\quad \Delta x_{maxS} \cdot 10^4 = 21.898\ \mu m$

Error in the dipole edge angle - $\dfrac{\mathrm{GdL}_{maxS}}{B} = 0.688 \cdot deg$

The offset in sextupole makes angle: $\theta_{sx} := \dfrac{\mathrm{SdL}_x \cdot \Delta x_{maxS}{}^2 \cdot e_{conv}}{2 \cdot p} = 9.797 \times 10^{-8}$

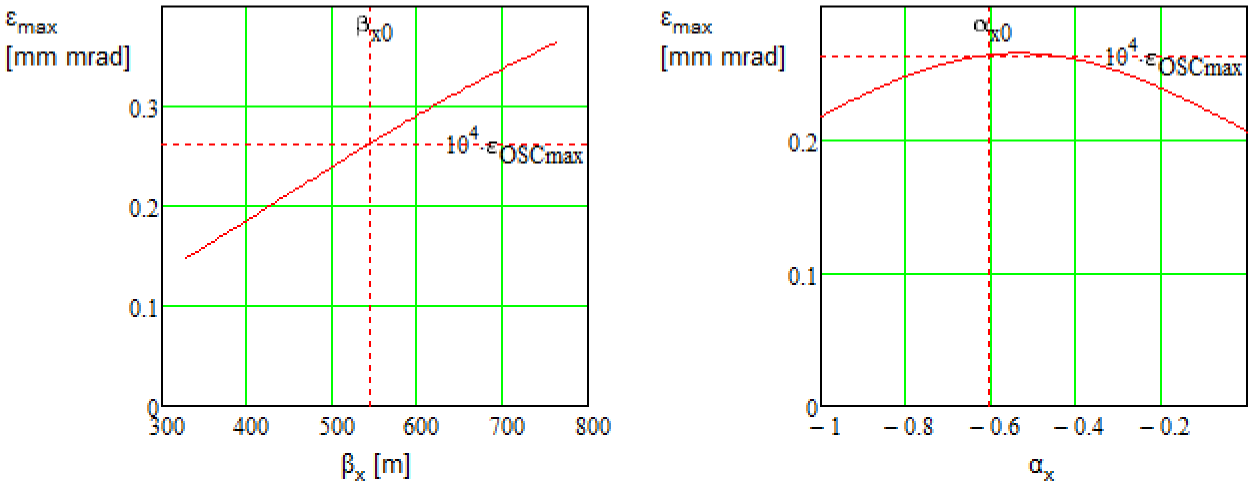
which corresponds to the beam displacement at beta-function of 5 m: $500 \cdot \theta_{sx} \cdot 10^4 = 0.49\ \ \mu m$

Sensitivity of OSC parameters to Optics Variations

- Sensitivity of cooling range to optics variations does not represent significant problems
- It requires
  - ♦ beta-function control <10%
  - ♦ Dispersion control <10 cm (<7% from maximum D)

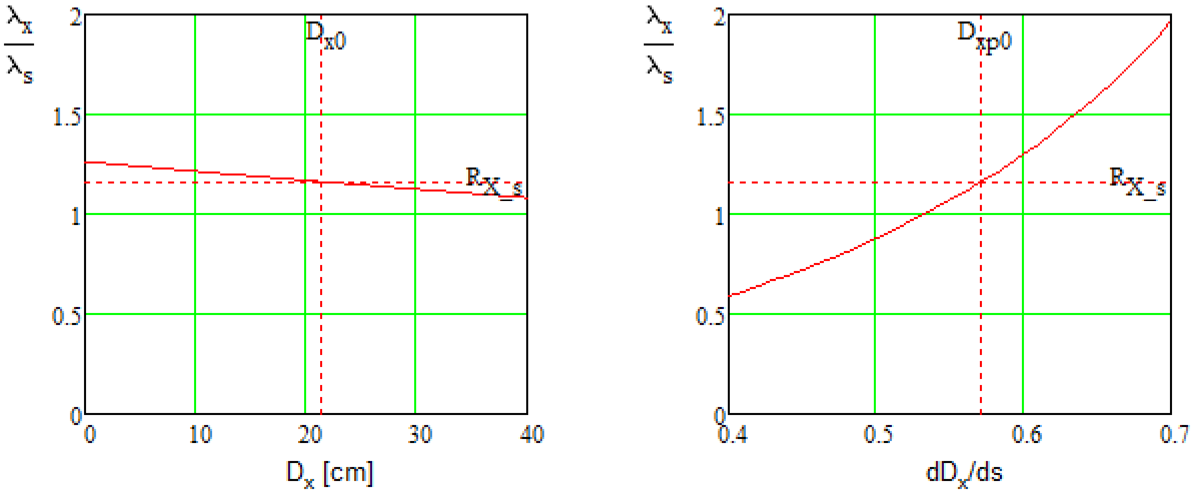
Effect of β-function variation on the cooling range

Effect of α-function variation on the cooling range



Effect of dispersion variation on the ratio of cooling rates

Effect of dispersion prime variation on the ratio of cooling

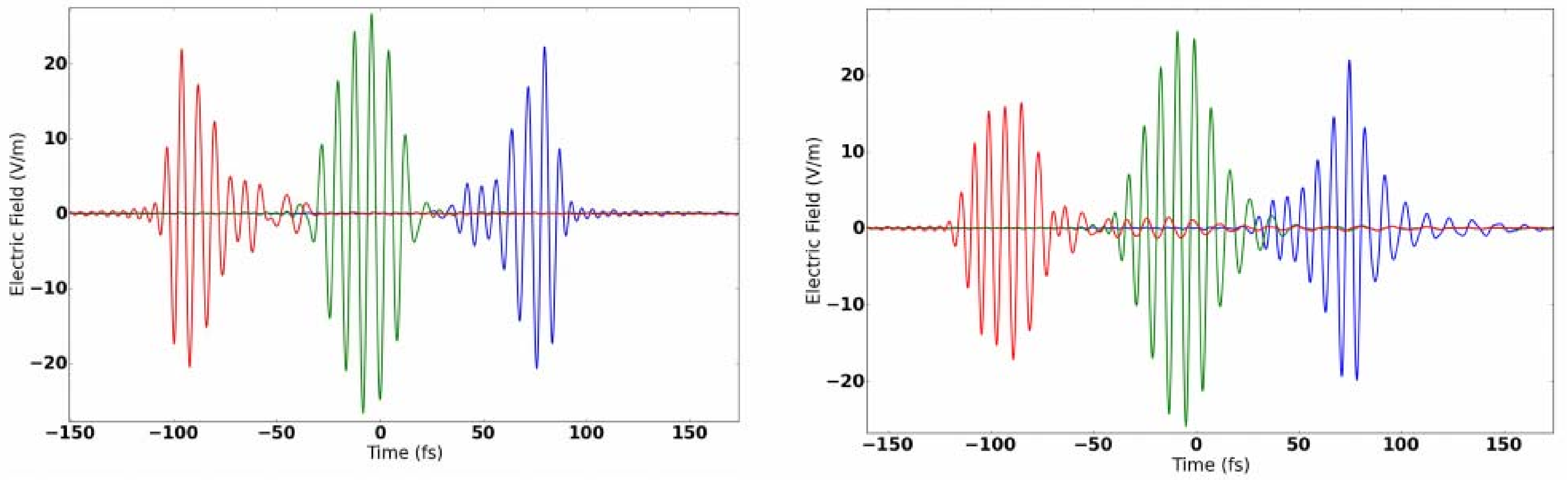


Dependence of cooling range and ratio of cooling rated on the beta-function and dispersion at the beginning of OSC section (starts at the end of pickup undulator)

Simulations with SRW (Synchrotron radiation workshop)

Jinhao Ruan and Matt Andorf

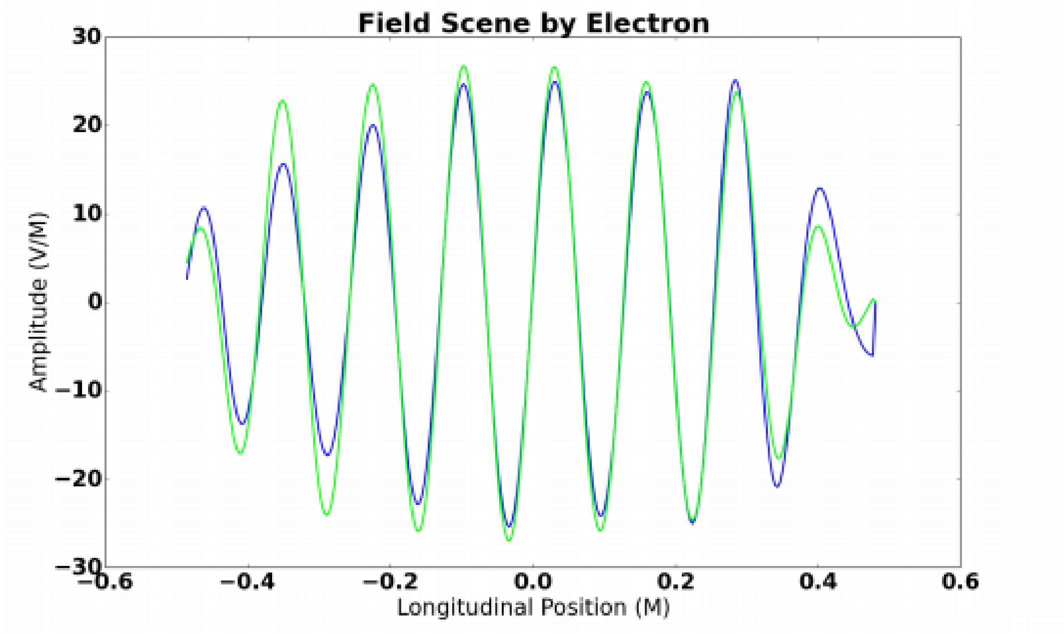
- SRW has an accurate model for SR and accounts for diffraction in the lenses and dispersion in their material
  - Particle interaction with e.-m. wave is accounted separately
    - Both transverse and longitudinal particle displacements are accounted
  - Good coincidence with previously derived analytical formulas
    - Simulations were helpful to understand details of interaction

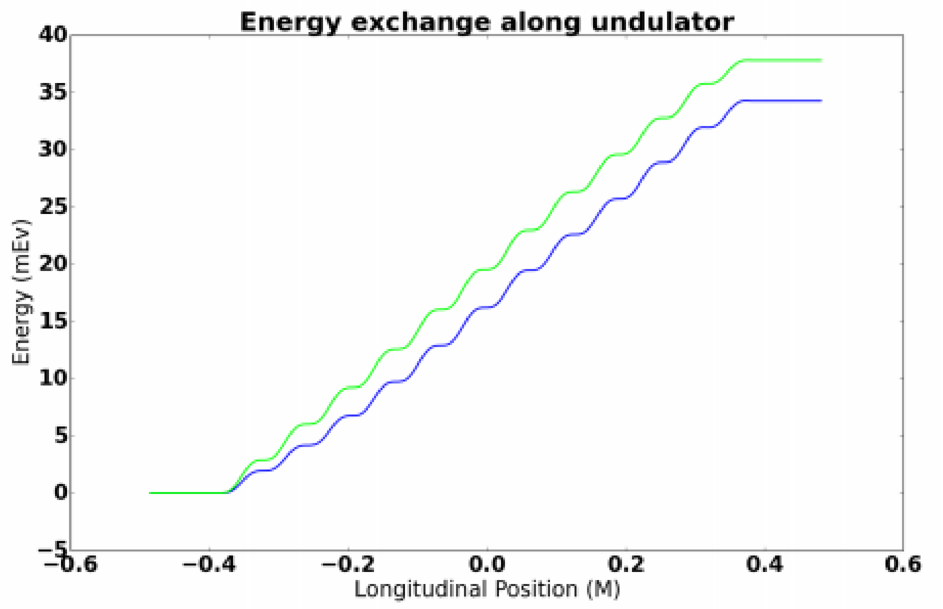


Light pulse at front,center and back of kicker. Left with no dispersion, right with dispersion.

Note a particle would move from left to right relative to light pulse with the way time field is plotted.

Simulations with SRW (continue)

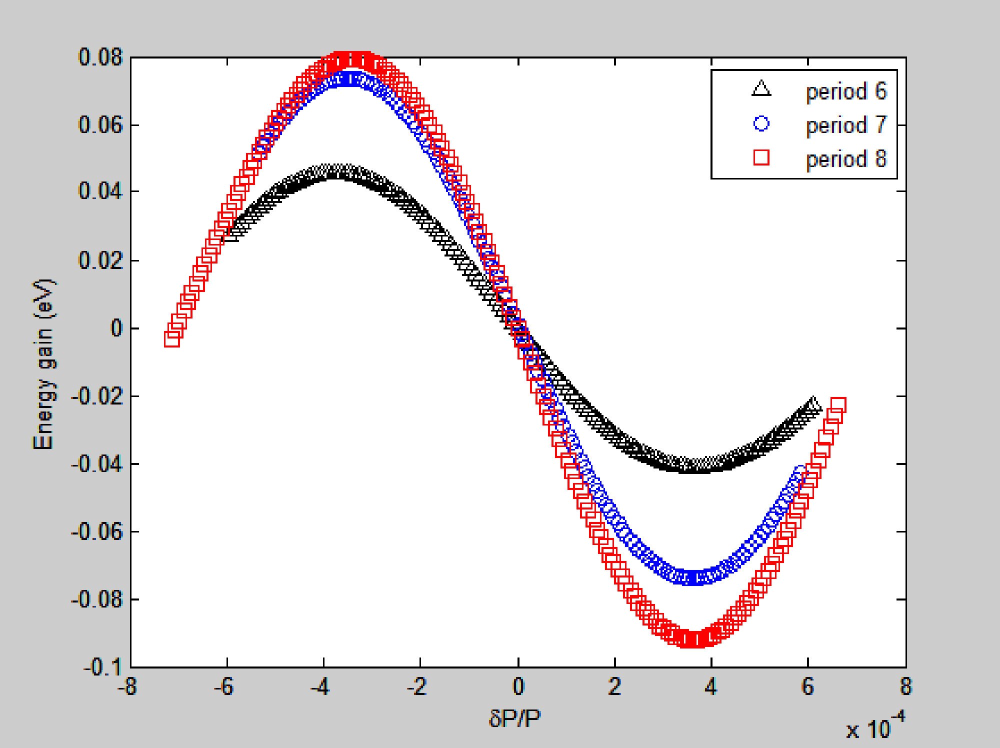




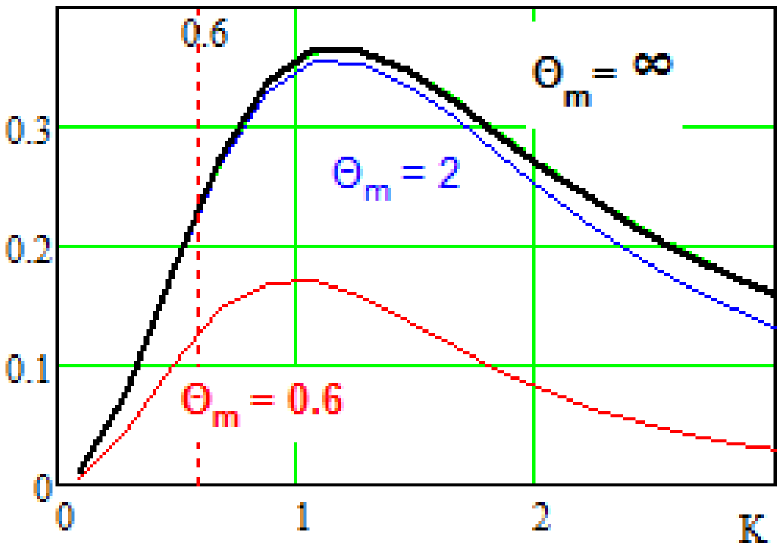
Green-no dispersion. Blue-dispersion
The effect of dispersion is light from front of pickup is not as focused as light from center/back.

Results in roughly 10% descrease in maximum kick.

Simulations with SRW (continue)



Energy loss estimate with different number of undulator periods. Total undulator length is fixed to about 75cm.



- Reduction of Cooling force with K_U is related to separation of radiation and particle due to motions in pickup and kicker undulators

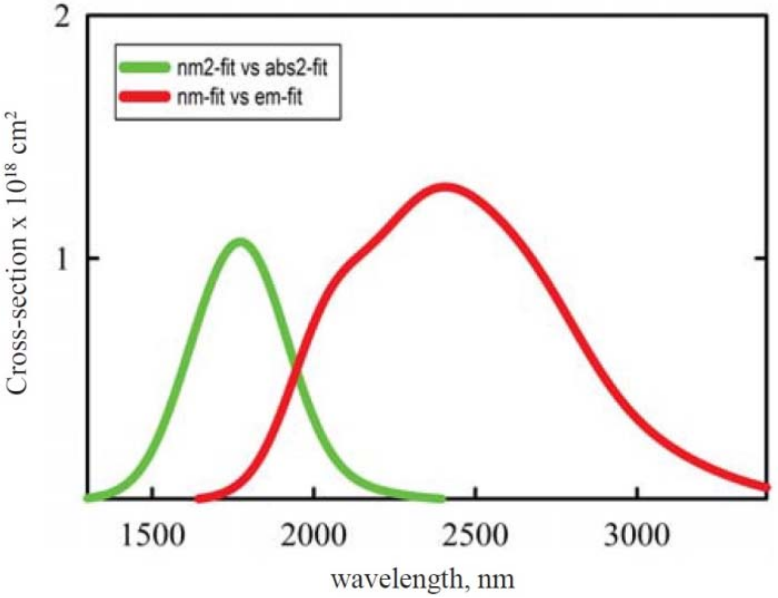
Single Pass Optical Amplifier for OSC at IOTA

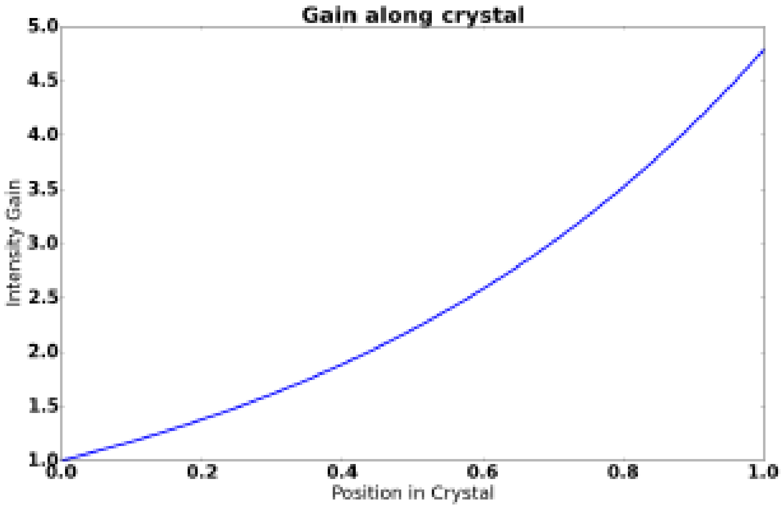
Matt Andorf and Philippe Piot

Basic Characteristics

- Cr:ZnSe solid state lasing gain medium.
- Bandwidth FWHM 2.2-2.9 μm.
- 1 mm length (~1.44 mm delay) .
- CW pumping at 1.93 μm with ~100 kW/cm^2

- Pump wavelength chosen because
    - High power (50-100 W) commercially available Thulium pump
    - Reduction in heat deposited in crystal over shorter wavelengths
- Gain
    - Combination of short crystal length, small signal intensity and depleted ground state gives rise to exponential signal growth through the crystal.
    - **Total gain in power, G= 5**





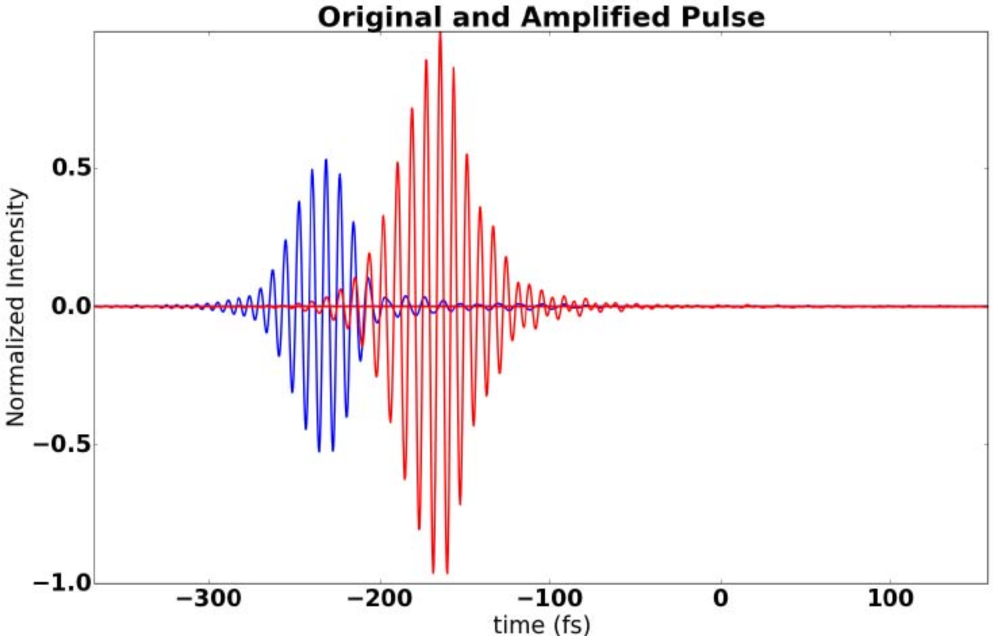
Single Pass Optical Amplifier for OSC at IOTA (2)

- The broadband pulse is modified in 3 ways while passing through the amplifier

$$E_2(\omega, z) = E_1(\omega)\exp[i(z\beta + \phi_{amp})]G^{\frac{1}{2(1+\Delta x^2)}}$$

  - Group Velocity Dispersion (GVD) from the host medium lengthens the pulse and introduces energy chirp, $\beta = 2\pi n/\lambda$
  - Gain narrowing (pulse broadening) from finite amplifier bandwidth
  - Phase distortions from amplification.
- Lengthening through GVD has largest effect, works to reduce field amplitude.

$$\gamma_{12}(\tau) = \frac{\langle E_1(t)E_2^*(t+\tau)\rangle}{\left[\langle|E_1|^2\rangle\langle|E_2|^2\rangle\right]^{1/2}}$$


Original and Amplified Pulse

- Correlation function multiplied by gain estimates total increase in kick

$$\gamma_{12}\sqrt{G} = 2.05$$

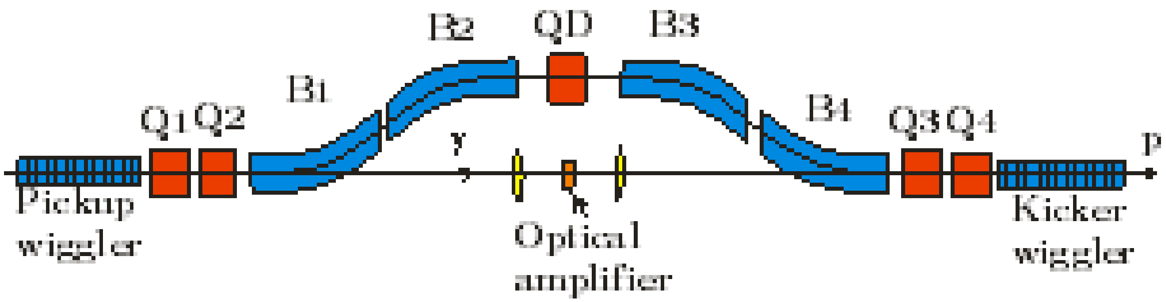
- **Amplifier increases damping rates by a factor of 2**

Conclusions

- Conceptual design of OSC experiment is close to be finished
- Writing Design Report is initiated
- We need to start the design of OSC chicane
  - Magnetic and vacuum designs are interdependent
    - It requires careful oversight of the design work
- Better understanding of OSC instrumentation is required
- Development of Optical Amplifier is important part of the work
  - Coherent efforts are required to verify its operation and usefulness for OSC

Backup Slides

OSC Limitations on IOTA Optics



- In the first approximation the orbit offset in the chicane (h), the path lengthening (δs) and the defocusing strength of chicane quad (Φ) together with dispersion and beta-function in the chicane center (D^*, β^*) and determine the entire cooling dynamics
- δs is set by delay in the amplifier => M_{56}
- $\Phi D^* h$ is determined by the ratio of decrements => for known ε we obtain the dispersion invariant (A^*)
- An average value of A in dipoles determines the equilibrium emittance. A^* is large and A needs to be reduced fast to get an acceptable value of the emittance (ε)

$$M_{56} \approx 2\Delta s\,,$$

$$\tilde{M}_{56} \approx 2\Delta s - \Phi D^* h\,,$$

$$\lambda_x / \lambda_s \approx \Phi D^* h / \left(2\Delta s - \Phi D^* h\right),$$

$$n_{\sigma p} \approx \mu_0 / \left(\left(2\Delta s - \Phi D^* h\right) k \sigma_p\right),$$

$$n_{\sigma x} \approx \mu_0 / \left(2kh\Phi\sqrt{\varepsilon\beta^*}\right),$$

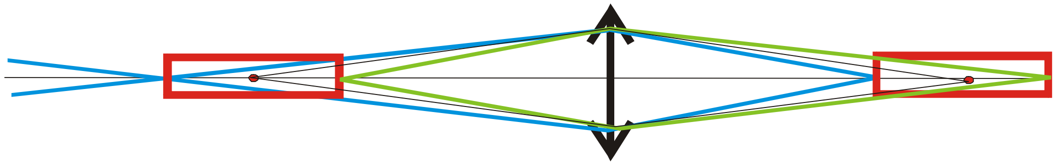
$$\Rightarrow \Phi D^* h \approx \frac{\mu_0}{2kn_{\sigma x}}\sqrt{\frac{A^*}{\varepsilon}}\,,\quad A^* \equiv \frac{D^{*2}}{\beta^*}$$

Parameters of Chicane Optics

Optics structure for half of the chicane staring from its center

N	Name	S[cm]	L[cm]	B[kG]	G[kG/cm]	S[kG/cm/cm]	Tilt[deg]
1	qqx1h	3	3	0	-0.075	0	0
2	oLX1L	8	5				
3	qqx2l	14	6	0	0	0	0
4	oLX2L	21	7				
5	gINbx1l	21	0	-2.498	Angle[deg]=0	Eff.Length[cm]=1	Tilt[deg]=0
6	bbx1l	29	8	-2.498	0 0 0 0	-3.41521	
7	gOUTbx1l	29	0	-2.498	Angle[deg]=3.415	Eff.Length[cm]=1	Tilt[deg]=0
8	oLX3L	39	10				
9	ssx1l	49	10	0	0	-0.75	0
10	oLX4L	60	11				
11	ssx2l	70	10	0	0	1.37	0
12	oLX5L	79.95	9.95				
13	gINbx2l	79.95	0	2.498	Angle[deg]=3.415	Eff.Length[cm]=1	Tilt[deg]=0
14	bbx2l	87.95	8	2.498	0 0 0 0	3.41521	
15	gOUTbx2l	87.95	0	2.498	Angle[deg]=0	Eff.Length[cm]=1	Tilt[deg]=0
16	oLX6L	94.95	7				
17	qqx3l	104.95	10	0	0.761025	0	0
18	oLX7L	111.95	7				
19	qqx4l	121.95	10	0	-0.513891	0	0
20	oLX8L	134.598	12.648				
21	bbwph	136.21	(undulator start)				

Basics of OSC – Correction of the Depth of Field

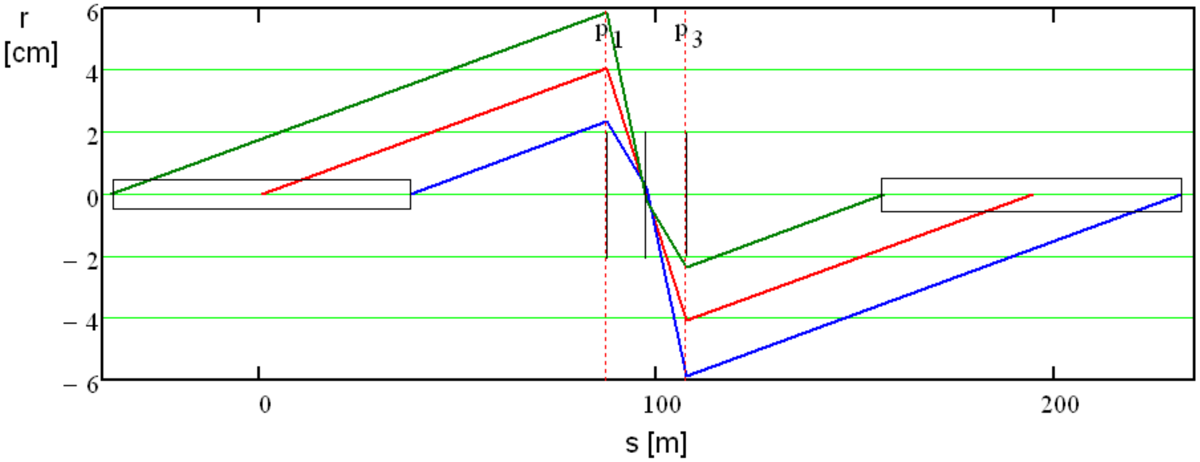


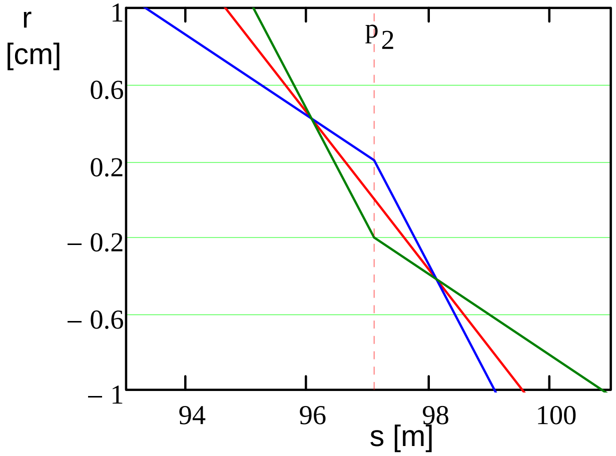
- It was implied above that the radiation coming out of the pickup undulator is focused on the particle during its trip through the kicker undulator
  - It can be achieved with lens located at infinity

$$\frac{1}{2F+\Delta s}+\frac{1}{2F-\Delta s}=\frac{1}{F}\quad\to\quad\frac{1}{F-\Delta s^2/4F}=\frac{1}{F}\quad\xrightarrow{F\to\infty}\quad\frac{1}{F}=\frac{1}{F}$$

  - but this arrangement cannot be used in practice
- A 3-lens telescope can address the problem within limited space

$$\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -F_2^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$





Choice of Optical Lens Material

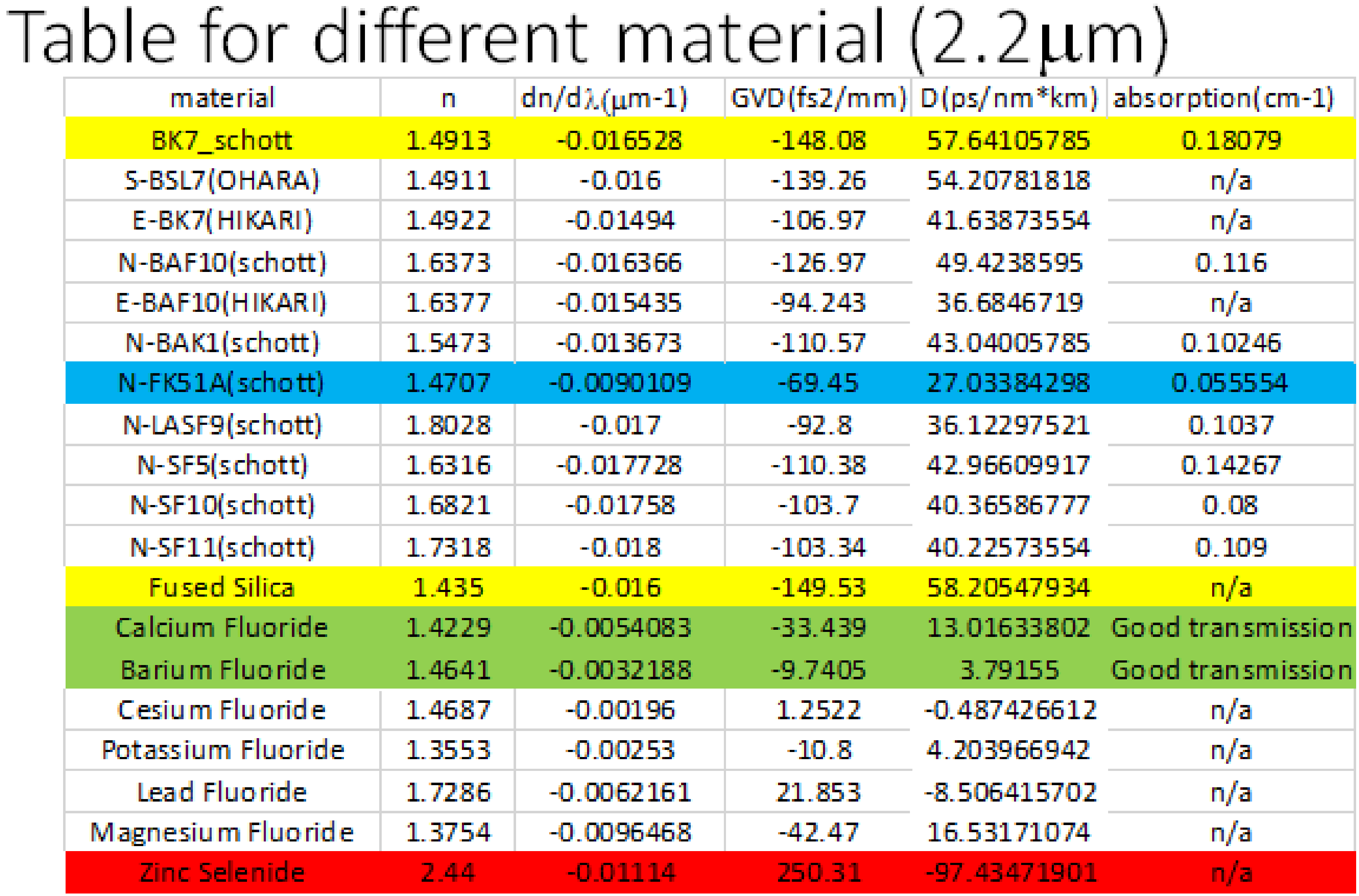
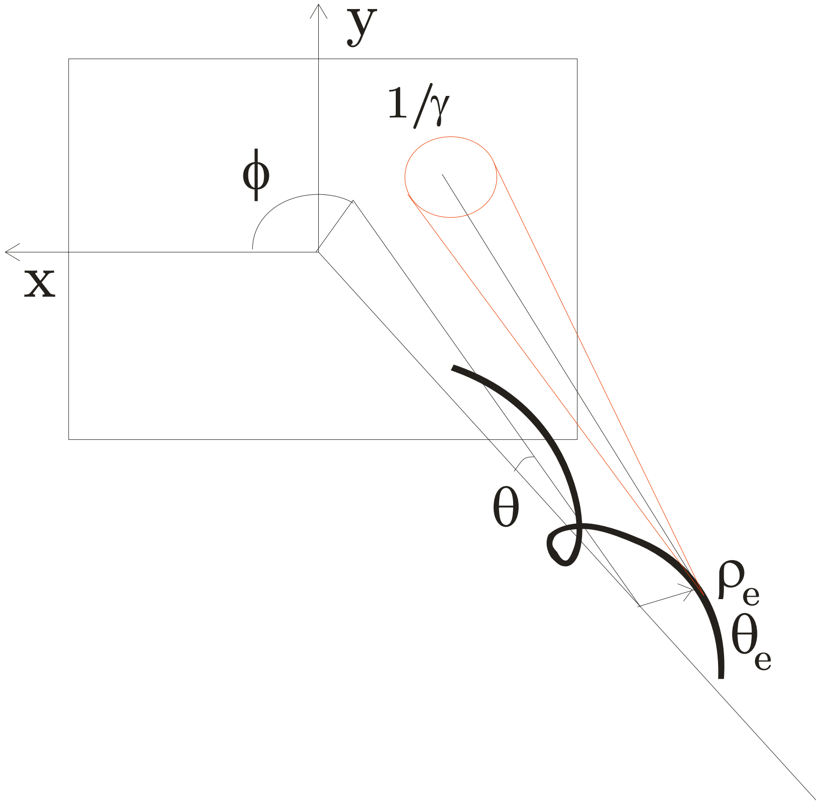


Table for different material (2.2μm)

material	n	dn/dλ(μm-1)	GVD(fs2/mm)	D(ps/nm*km)	absorption(cm-1)
BK7_schott	1.4913	-0.016528	-148.08	57.64105785	0.18079
S-BSL7(OHARA)	1.4911	-0.016	-139.26	54.20781818	n/a
E-BK7(HIKARI)	1.4922	-0.01494	-106.97	41.63873554	n/a
N-BAF10(schott)	1.6373	-0.016366	-126.97	49.4238595	0.116
E-BAF10(HIKARI)	1.6377	-0.015435	-94.243	36.6846719	n/a
N-BAK1(schott)	1.5473	-0.013673	-110.57	43.04005785	0.10246
N-FK51A(schott)	1.4707	-0.0090109	-69.45	27.03384298	0.055554
N-LASF9(schott)	1.8028	-0.017	-92.8	36.12297521	0.1037
N-SF5(schott)	1.6316	-0.017728	-110.38	42.96609917	0.14267
N-SF10(schott)	1.6821	-0.01758	-103.7	40.36586777	0.08
N-SF11(schott)	1.7318	-0.018	-103.34	40.22573554	0.109
Fused Silica	1.435	-0.016	-149.53	58.20547934	n/a
Calcium Fluoride	1.4229	-0.0054083	-33.439	13.01633802	Good transmission
Barium Fluoride	1.4641	-0.0032188	-9.7405	3.79155	Good transmission
Cesium Fluoride	1.4687	-0.00196	1.2522	-0.487426612	n/a
Potassium Fluoride	1.3553	-0.00253	-10.8	4.203966942	n/a
Lead Fluoride	1.7286	-0.0062161	21.853	-8.506415702	n/a
Magnesium Fluoride	1.3754	-0.0096468	-42.47	16.53171074	n/a
Zinc Selenide	2.44	-0.01114	250.31	-97.43471901	n/a

Basics of OSC – Radiation from Undulator



- Liénard-Wiechert potentials and E-field of moving charge in wave zone

$$\begin{cases} \varphi(\mathbf{r},t) = \left.\dfrac{e}{\left(R - \boldsymbol{\beta}\cdot\mathbf{R}\right)}\right|_{t-R/c} \\ \mathbf{A}(\mathbf{r},t) = \left.\dfrac{e\mathbf{v}}{\left(R - \boldsymbol{\beta}\cdot\mathbf{R}\right)}\right|_{t-R/c} \end{cases} \Rightarrow$$

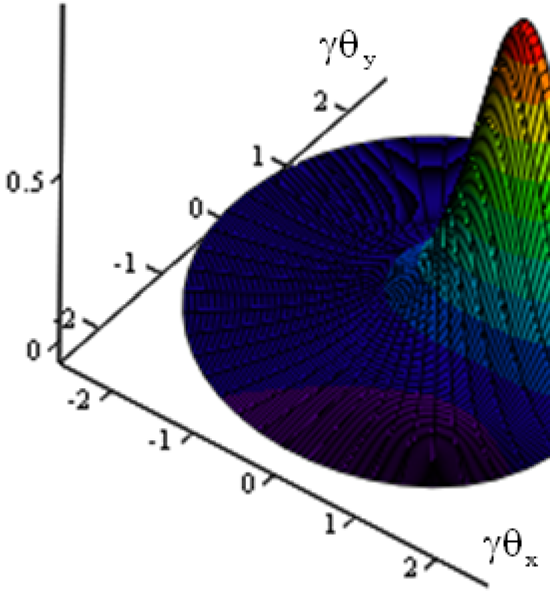
$$\mathbf{E}(\mathbf{r},t) = \frac{e}{c^2}\left.\frac{\left(\mathbf{R} - \boldsymbol{\beta}\cdot R\right)\left(\mathbf{a}\cdot\mathbf{R}\right) - \mathbf{a}R\left(R - \boldsymbol{\beta}\cdot\mathbf{R}\right)}{\left(R - \boldsymbol{\beta}\cdot\mathbf{R}\right)^3}\right|_{t-R/c}$$

where $\mathbf{a} \equiv \dfrac{d\mathbf{v}}{dt}$

- Radiation of ultra-relativistic particle is concentrated in 1/γ angle
- Undulator parameter:

$$K \equiv \gamma\theta_e = \frac{\lambda_{wgl}}{2\pi}\frac{eB_0}{mc^2}$$

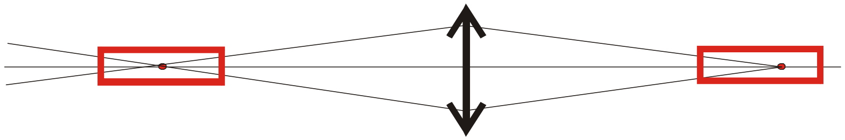
- For K ≥ 1 the radiation is mainly radiated into higher harmonics



E_x for K=1

Basics of OSC – Radiation Focusing to Kicker Undulator

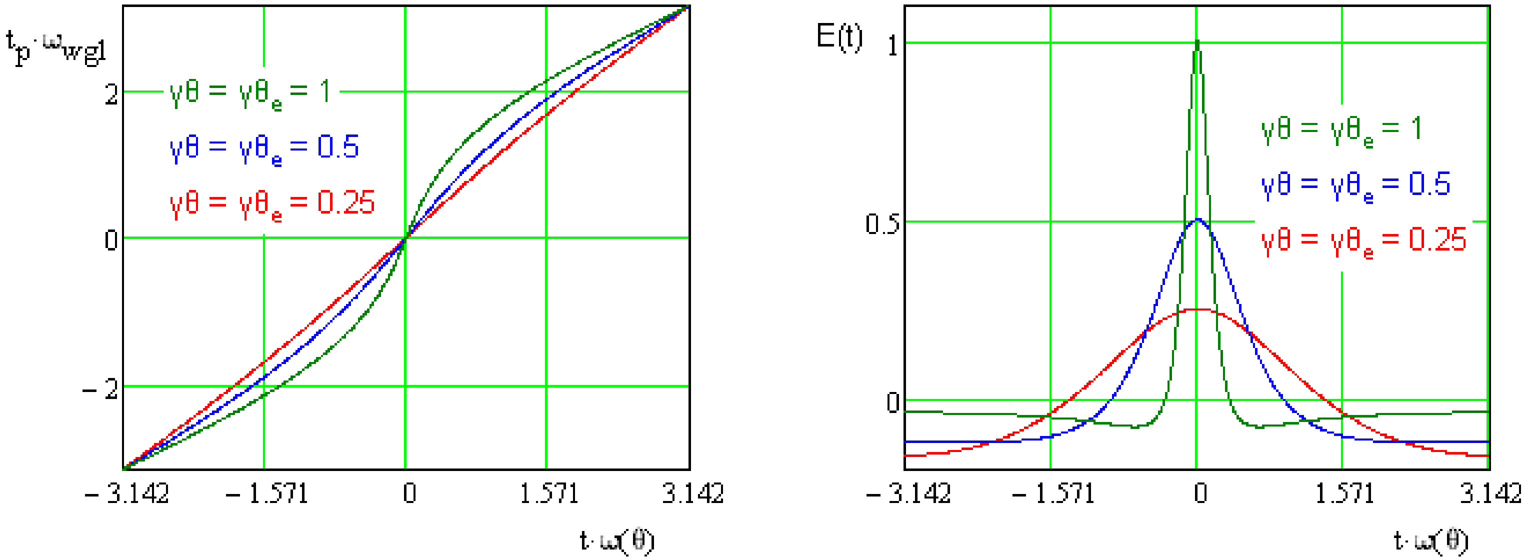
- Modified Kirchhoff formula



$$E(r)=\frac{\omega}{2\pi i c}\int_S \frac{E(r')}{|r-r'|}\mathrm{e}^{i\omega|r-r'|}\,ds'$$

=> $$E(r)=\frac{1}{2\pi i c}\int_S \frac{\omega(r')E(r')}{|r-r'|}\mathrm{e}^{i\omega|r-r'|}\,ds'$$

- Effect of higher harmonics
  - Higher harmonics are normally located outside window of optical lens transparency and are absorbed in the lens material

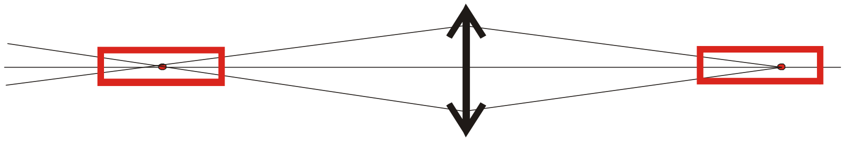


Dependences of retarded time (t_p) and E_x on time for helical undulator

- Only first harmonic is retained in the calculations presented below

Basics of OSC – Longitudinal Kick for K<<1

- For $K << 1$ refocused radiation of pickup undulator has the same structure as radiation from kicker undulator. They are added coherently:

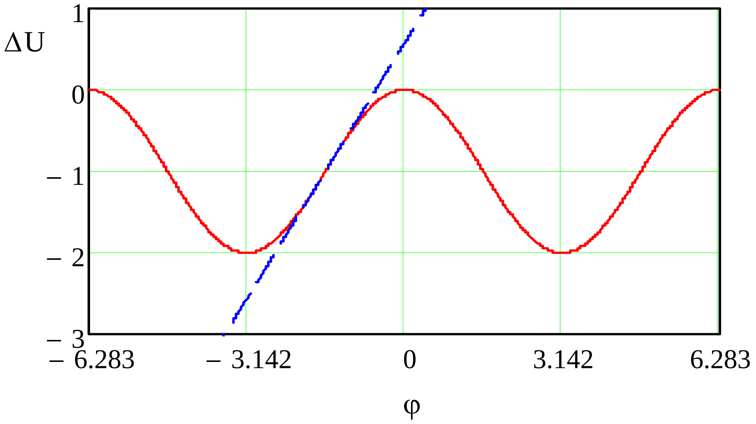


$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 e^{i\phi} \xrightarrow{\mathbf{E}_1=\mathbf{E}_2} 2\cos\left(\phi/2\right)\mathbf{E}_1 e^{i\phi/2}$$

⇨ Energy loss after passing 2 undulators

$$\Delta U \propto \left|E^2\right| = 4\cos\left(\phi/2\right)^2\left|\mathbf{E}_1{}^2\right| = 2\left(1+\cos\phi\right)\left|\mathbf{E}_1{}^2\right| = 2\left(1+\cos\left(kM_{56}\frac{\Delta p}{p}\right)\right)\left|\mathbf{E}_1{}^2\right|$$

- Large derivative of energy loss on momentum amplifies damping rates and creates a possibility to achieve damping without optical amplifier



  - SR damping: $\lambda_{\|\_SR} \approx \dfrac{2\Delta U_{SR}}{pc} f_0$
  - OSC: $\lambda_{\|\_OSC} \approx f_0 \dfrac{2\Delta U_{wgl}}{pc}\left(GkM_{56}\right) \xrightarrow{kM_{56}\left(\Delta p/p\right)_{\max}=\pi} f_0 \dfrac{2\Delta U_{wgl}}{pc}\left(\dfrac{G}{\left(\Delta p/p\right)_{\max}}\right)$

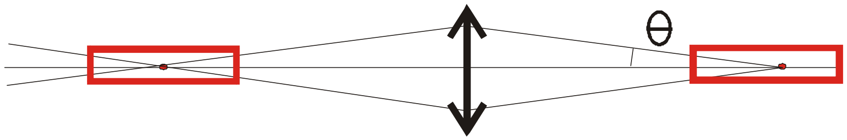
    where G - optical amplifier gain, $(\Delta p/p)_{max}$ - cooling system acceptance

⇨ $\lambda_{\|\_OSC} \propto B^2 L \propto K^2 L$ - but cooling efficiency drops with K increase above ~1

Basics of OSC – Longitudinal Kick for K<<1(continue)

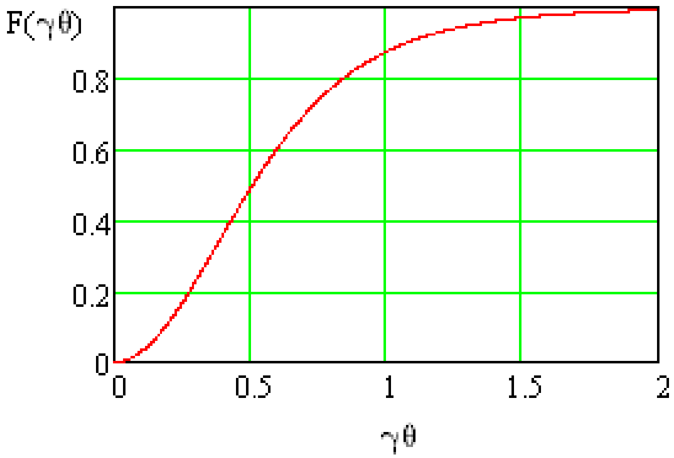
- Radiation wavelength depends on θ as

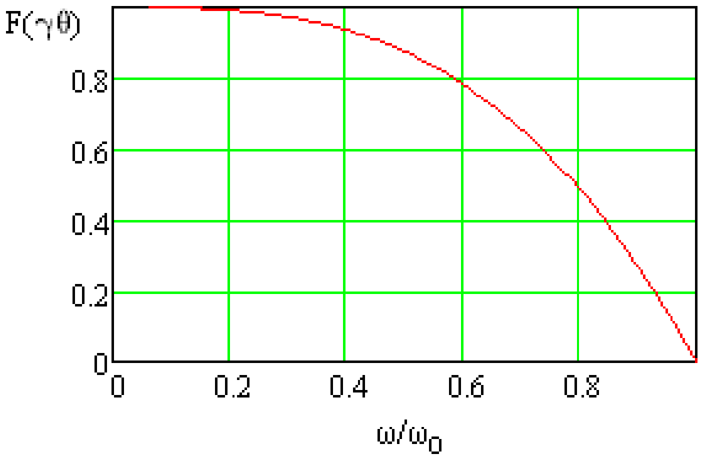
$$\lambda = \frac{\lambda}{2\gamma^2}\left(1+\gamma^2\theta^2\right)$$



Limitation of system bandwidth by (1) optical amplifier band or (2) subtended angle reduce damping rate

$$\lambda_{\|\_SR} = \lambda_{\|\_SR0} F\left(\gamma\theta_{\mathrm{m}}\right), \qquad F\left(x\right) = 1 - \frac{1}{\left(1+x^2\right)^3}$$





- For narrow band: $\Delta U_{wgl} = \Delta U_{wgl0}\left(\frac{3\Delta\omega}{\omega}\right), \quad \frac{3\Delta\omega}{\omega} << 1$

where $\Delta U_{wgl0} = \frac{e^4B^2\gamma^2L}{3m^2c^4}\begin{cases}1, & \text{Flat wiggler}\\ 2, & \text{Helical wiggler}\end{cases}$ the energy radiated in one undulator

Basics of OSC – Radiation from Flat Undulator

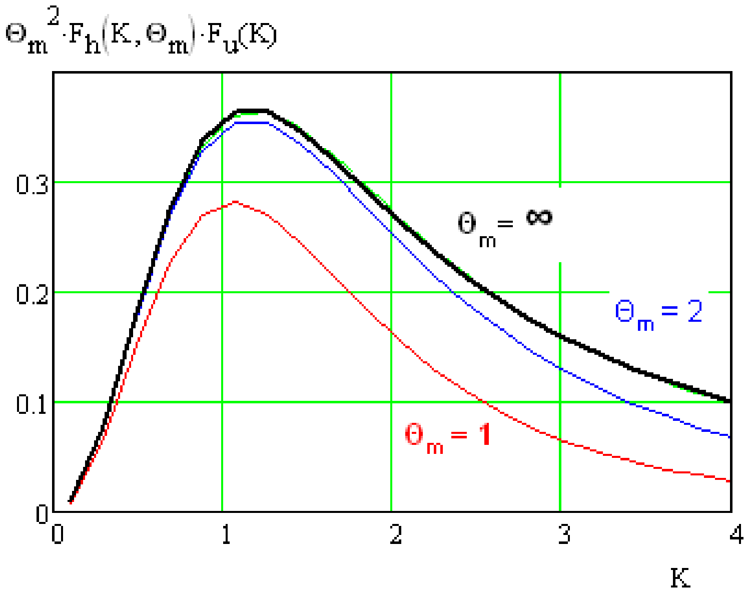
- For arbitrary undulator parameter we have

$$\Delta U_{OSC\_F} = \frac{1}{2}\frac{4e^4 B_0^{\ 2}\gamma^2 L}{3m^2c^4} G F_f\left(K, \gamma\theta_{\max}\right) F_u\left(\kappa_u\right)$$

$$F_u\left(\kappa_u\right) = \mathrm{J}_0\left(\kappa_u\right) - \mathrm{J}_1\left(\kappa_u\right), \quad \kappa_u = K^2 / \left(4\left(1 + K^2/2\right)\right)$$

Fitting results of numerical integration yields:

$$F_h\left(K,\infty\right) \approx \frac{1}{1 + 1.07K^2 + 0.11K^3 + 0.36K^4}, \quad K \equiv \gamma\theta_e \le 4$$



- Dependence of wave length on θ:

$$\lambda \approx \frac{\lambda_{wgl}}{2\gamma^2}\left(1 + \gamma^2\left(\theta^2 + \frac{\theta_e^{\ 2}}{2}\right)\right)$$

$$K \equiv \gamma\theta_e$$

- Flat undulator is "more effective" than the helical one
- For the same K and λ_{wgl} flat undulator generates shorter wave lengths

- For both cases of the flat and helical undulators and for fixed *B* a decrease of λ_{wgl} and, consequently, λ yields kick increase
  - but wavelength is limited by both beam optics and light focusing